RAPTOR: a lightweight transport model for open-loop optimization and real-time simulation

Federico Felici

O. Sauter, S. Coda, B.P. Duval, T.P. Goodman J-M. Moret and the TCV team

CRPP-EPFL, Association Euratom-Suisse, CH-1015 Lausanne Switzerland

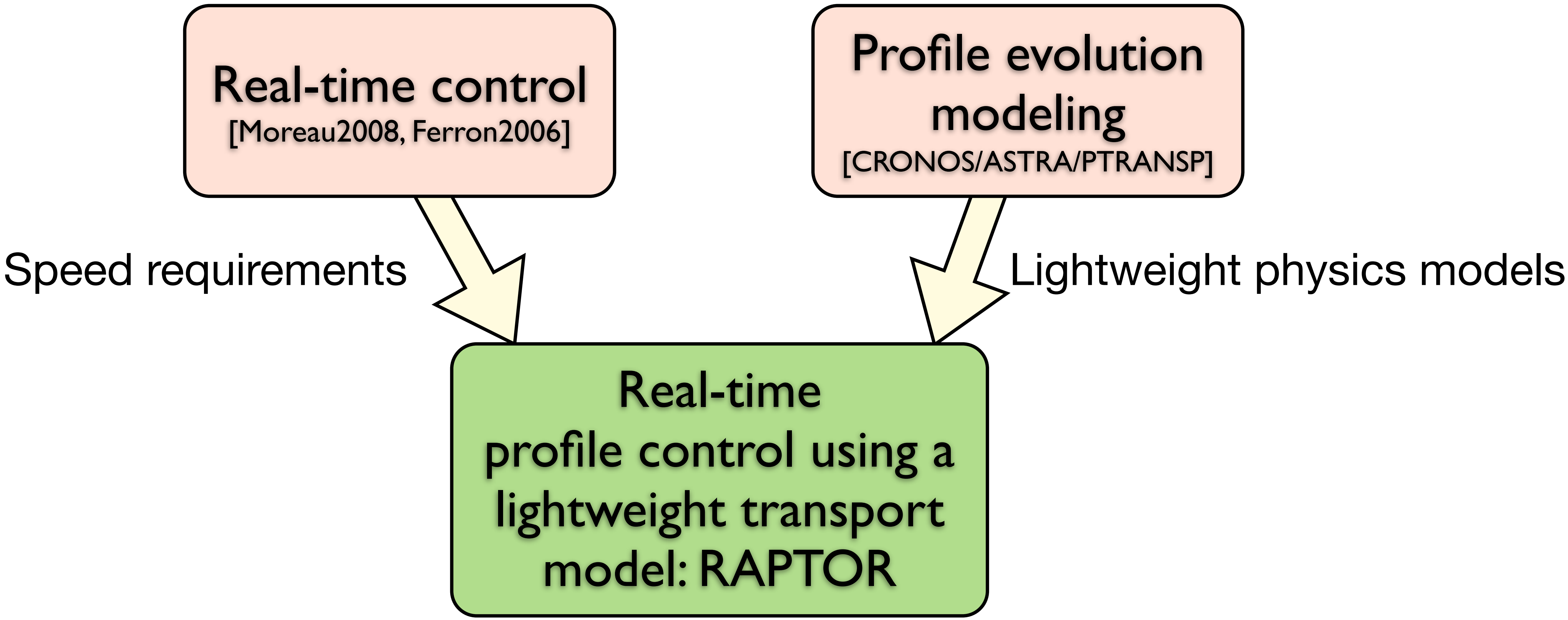
ITM-ISM working session 05.07.2011

Using transport models for profile control

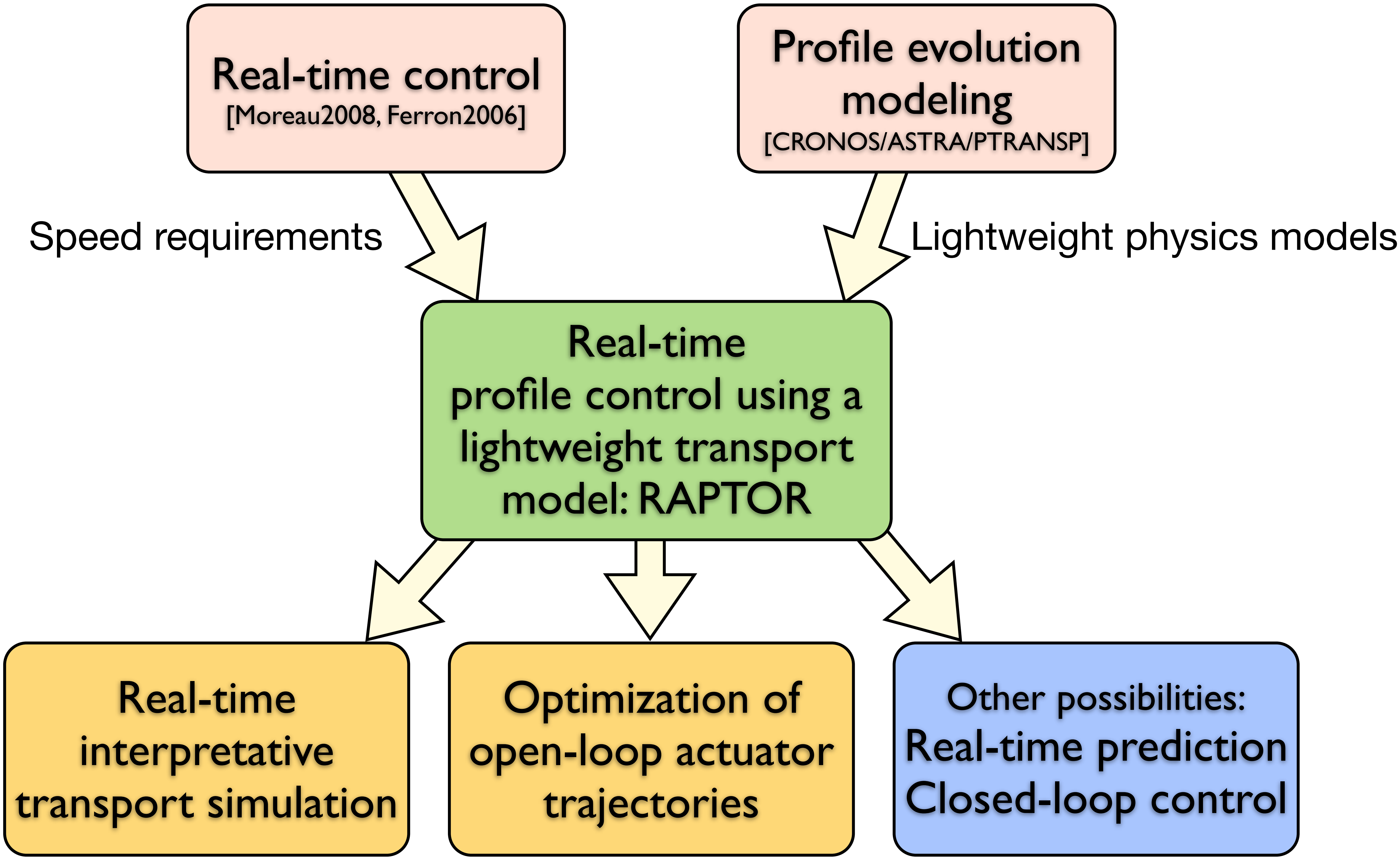
Real-time control
[Moreau2008, Ferron2006]

Profile evolution modeling
[CRONOS/ASTRA/PTRANSP]

Using transport models for profile control



Using transport models for profile control



Need lightweight transport model
Compatible with real-time execution

- **RAPTOR** - **RA**pid **P**lasma **T**ransport simulat**OR**

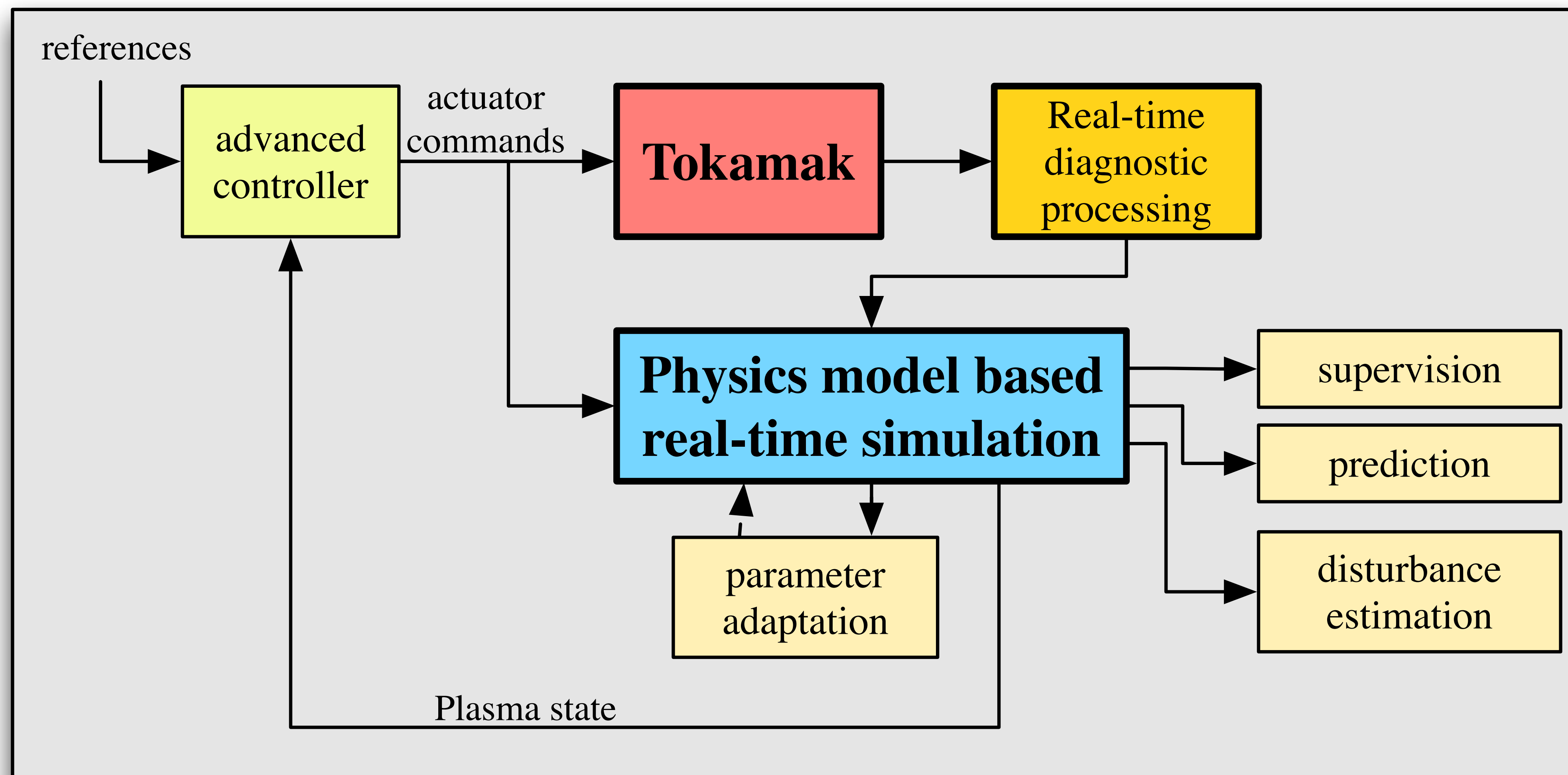
- Fast 1-D transport code for real-time implementation and fast optimization
  - Evolves profiles of poloidal flux $\psi(\rho,t)$, and electron temperature $T_e(\rho,t)$
  - Fixed flux surface shapes from pre-calculated MHD equilibrium
  - Neoclassical resistivity, bootstrap current [Sauter PoP 1999,2002]
  - q, shear profile dependent ad-hoc transport model χ_e (similar to [Polevoi2002,Garcia2010])
  - Parametrized heating / current drive sources

- Includes nonlinear profile coupling, crucial for hybrid/advanced scenarios

- Similar to [Witrant, PPCF 2007]
  - But additionally solves full T_e profile dynamics
  - Different numerics (Finite Elements, implicit solver)

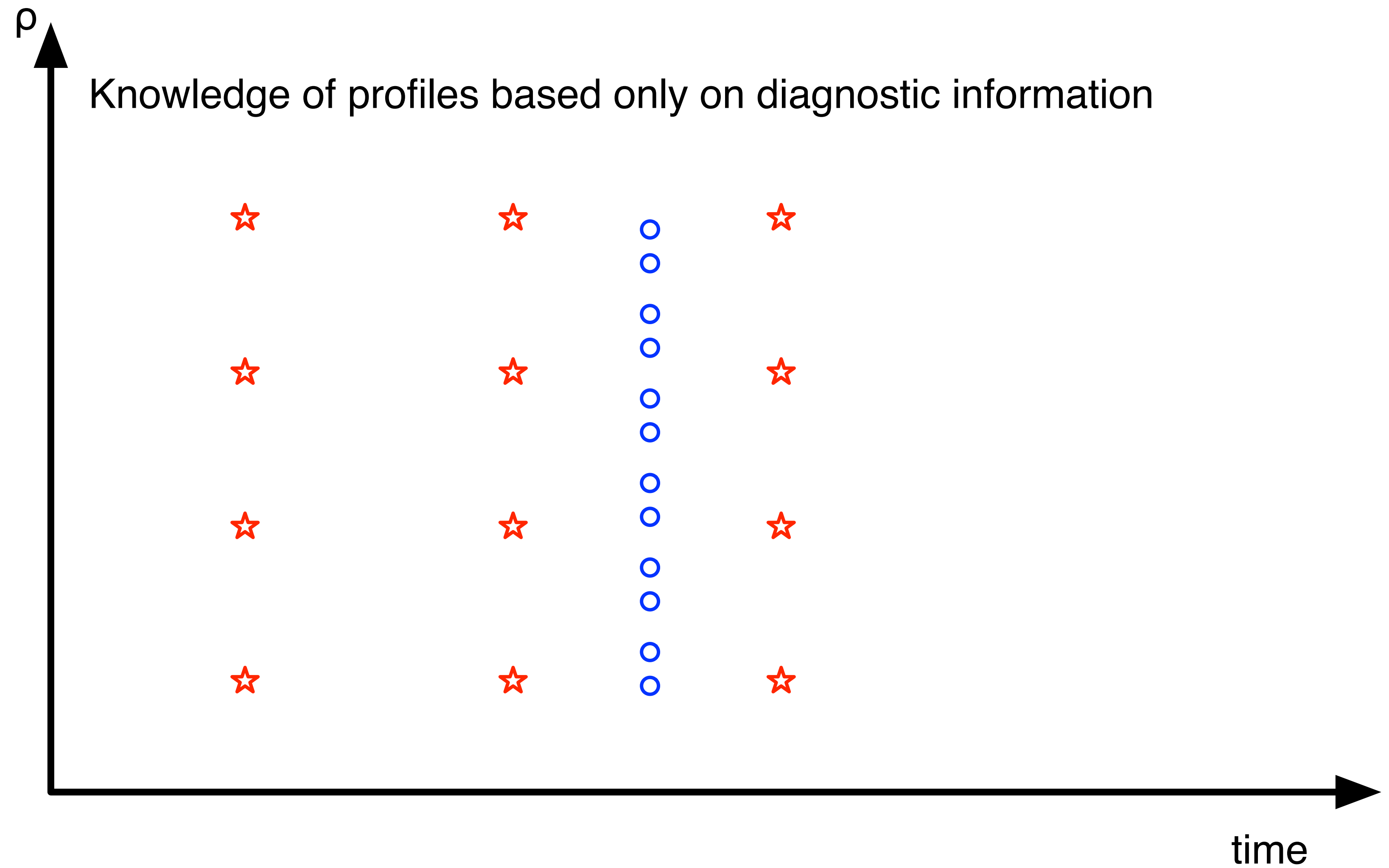
New paradigm: real-time simulation for physics-based plasma state reconstruction

- Real-time simulation
  - Evolve plasma numerically, while it is physically evolving in the tokamak
  - Use *available* diagnostics as constraints
  - In control engineering terms: a *Nonlinear, dynamic model-based state observer*
  - Model-reality mismatch: disturbance estimation or parameter adaptation



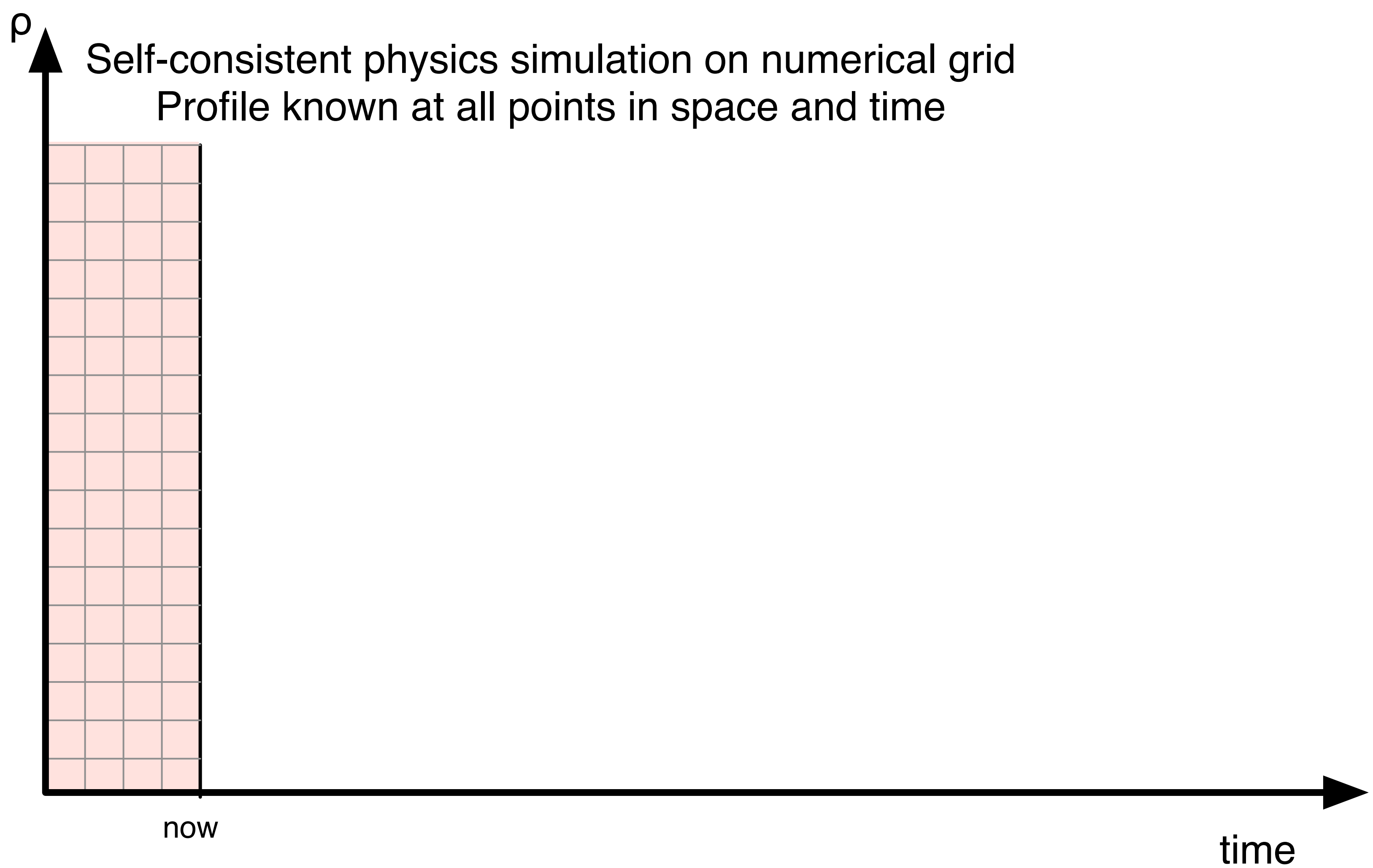
Real-time simulation: time/space resolution independent of diagnostic resolution

- Continuous function on (ρ,t)
- Different diagnostics give information at different spatial and temporal points
  - Today: profile information (e.g. for control) based exclusively on these points



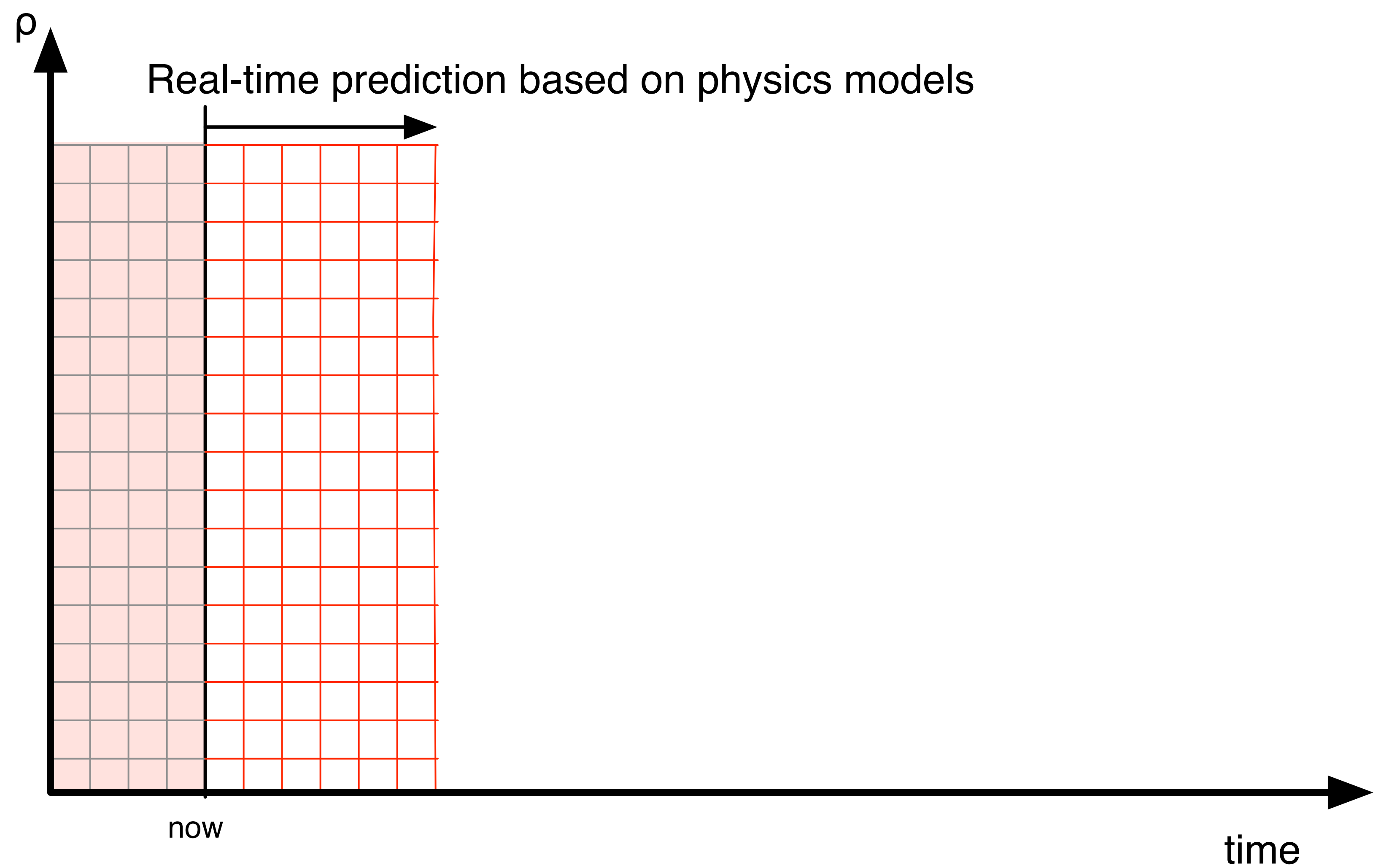
Real-time simulation: time/space resolution independent of diagnostic resolution

- Continuous function on (ρ,t)
- Different diagnostics give information at different spatial and temporal points
  - Today: profile information (e.g. for control) based exclusively on these points
- Real-time simulation of physical evolution done on independent numerical grid
  - Grid size determined by underlying physics, available CPU power



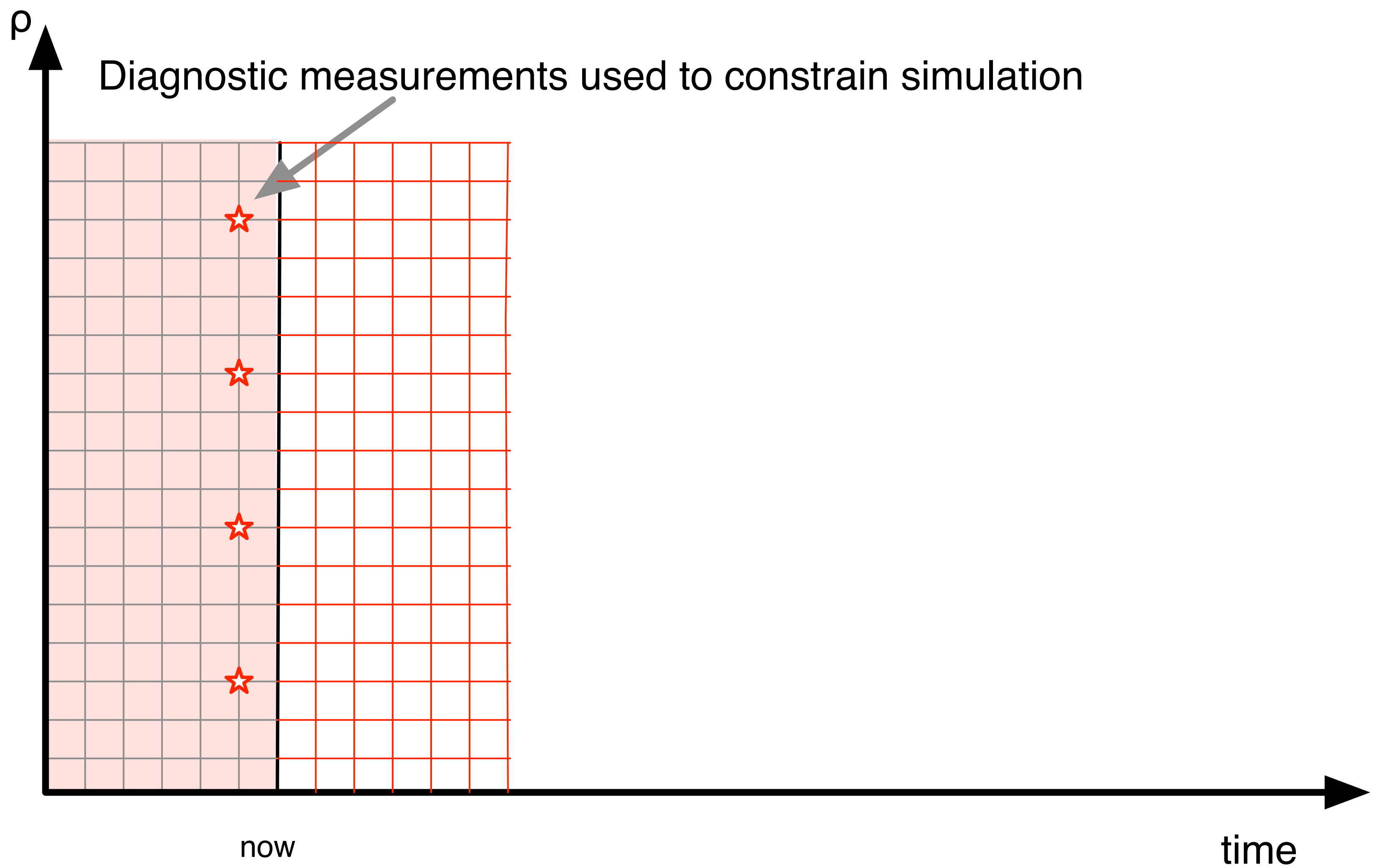
Real-time simulation: time/space resolution independent of diagnostic resolution

- Continuous function on (ρ,t)
- Different diagnostics give information at different spatial and temporal points
  - Today: profile information (e.g. for control) based exclusively on these points
- Real-time simulation of physical evolution done on independent numerical grid
  - Grid size determined by underlying physics, available CPU power



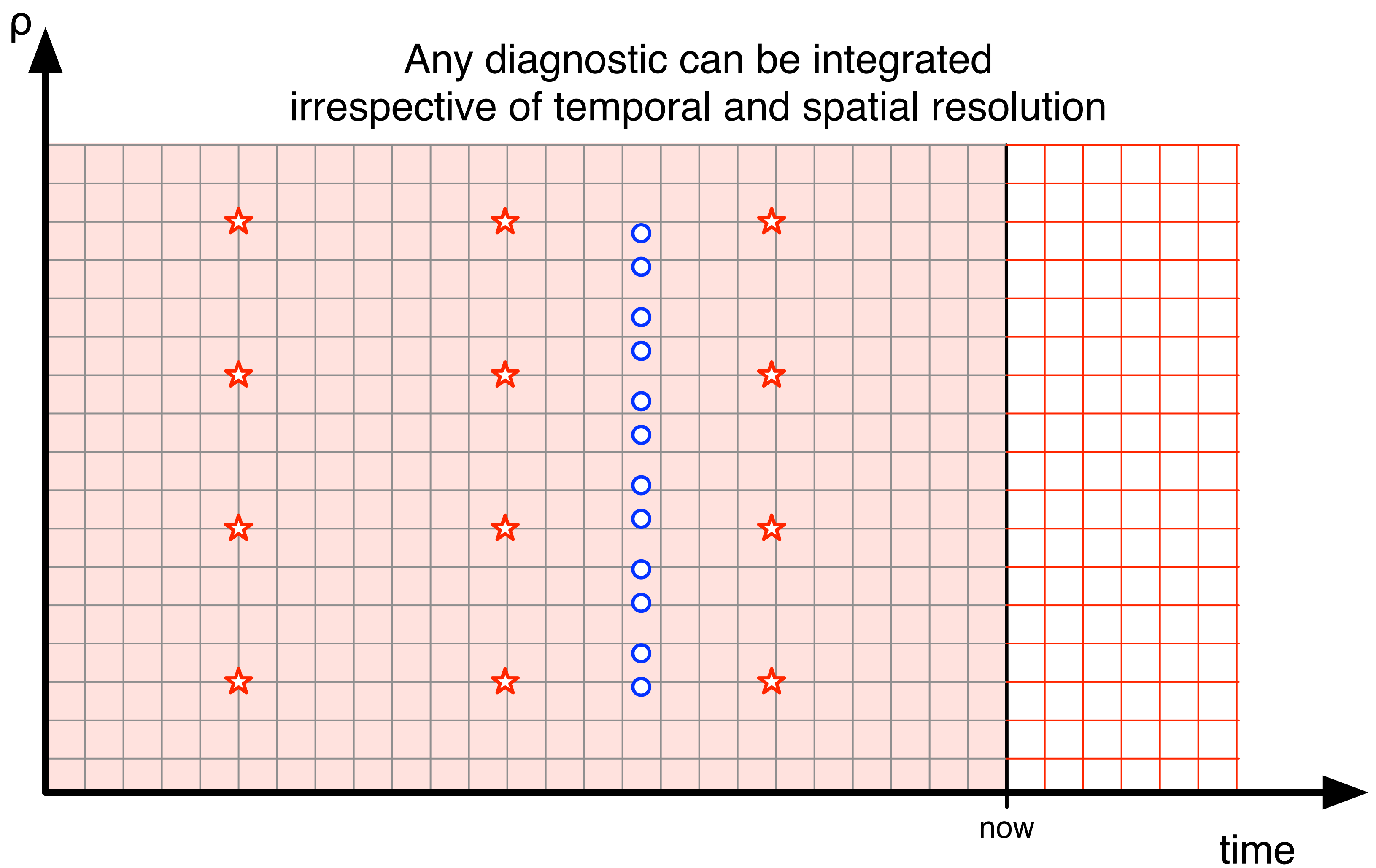
Real-time simulation: time/space resolution independent of diagnostic resolution

- Continuous function on (ρ,t)
- Different diagnostics give information at different spatial and temporal points
  - Today: profile information (e.g. for control) based exclusively on these points
- Real-time simulation of physical evolution done on independent numerical grid
  - Grid size determined by underlying physics, available CPU power
  - Diagnostics provide information to improve state estimation *when available*



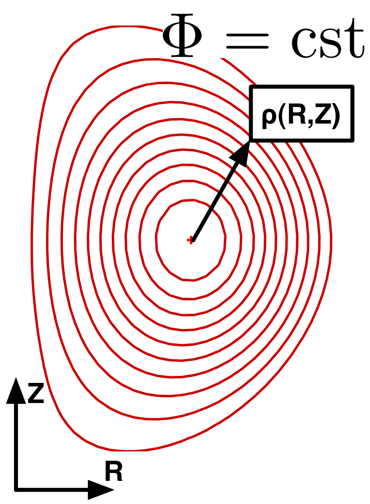
Real-time simulation: time/space resolution independent of diagnostic resolution

- Continuous function on (ρ,t)
- Different diagnostics give information at different spatial and temporal points
  - Today: profile information (e.g. for control) based exclusively on these points
- Real-time simulation of physical evolution done on independent numerical grid
  - Grid size determined by underlying physics, available CPU power
  - Diagnostics provide information to improve state estimation *when available*



Simplified physics model for real-time poloidal flux transport simulations

- Poloidal flux diffusion equation (1-D)
  - Solved assuming fixed flux surface distribution on (R,Z)



$$\sigma_{||}\frac{\partial\psi}{\partial t} = \frac{R_0 J^2}{\mu_0\rho}\frac{\partial}{\partial\rho}\left(\frac{G_2}{J}\frac{\partial\psi}{\partial\rho}\right) - \frac{V'}{2\pi\rho}(j_{BS}+j_{CD})$$

$$\rho = \sqrt{\frac{\Phi}{\pi B_0}},\quad J = \frac{RB_\phi}{R_0B_0},\quad V' = \frac{\partial V}{\partial\rho},\quad G_2 = \frac{V'}{4\pi^2}\left\langle\frac{(\nabla\rho)^2}{R^2}\right\rangle$$

[Hinton&Hazeltine Rev. Mod. Phys 1976], [Pereverzev IPP rep 1991]

- Sources

$$j_{BS} = -\frac{2\pi J(\psi)}{B_0 R_{pe}}\frac{\partial\rho}{\partial\psi}\left[\mathcal{L}_{31}\frac{\partial n_e}{\partial\rho}T_e + (\mathcal{L}_{31} + R_{pe}\mathcal{L}_{32} + (1-R_{pe})\alpha\mathcal{L}_{34})\frac{\partial T_e}{\partial\rho}n_e\right]$$

Bootstrap current: Neoclassical physics [Sauter PoP 1999]

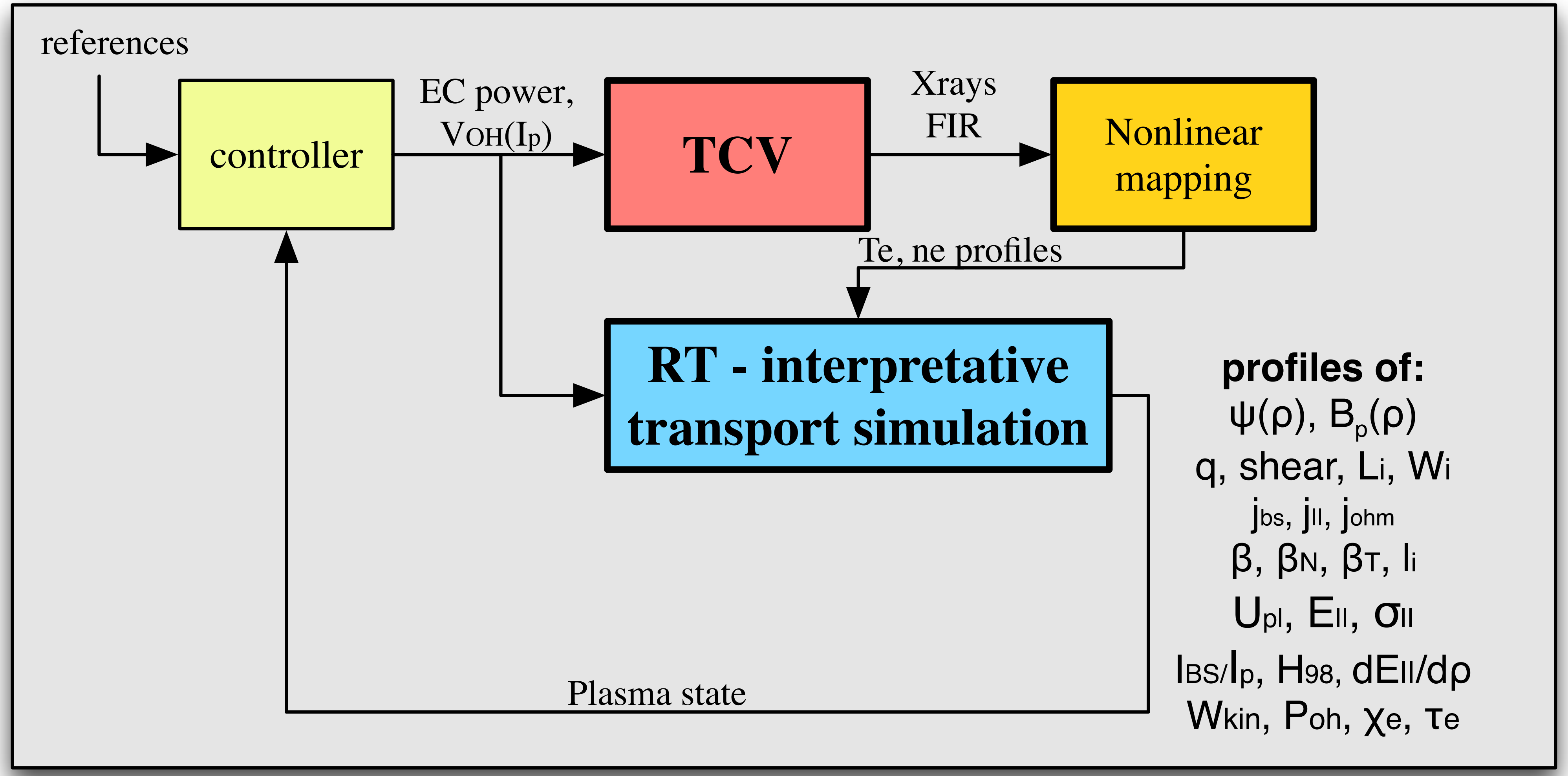
$$j_{ECCD}(\rho,t) = \underbrace{c_{exp}e^{-\rho^2/0.5^2}}_{\eta_{EC}}\frac{T_e}{n_e}\exp\left\{\frac{(\rho-\rho_{dep})^2}{w_{cd}^2}\right\}P_{gyro}(t)$$

Gaussian shape for EC current deposition

- Need inputs: I_p, $T_e(\rho)$, $n_e(\rho)$ at each time step: **from RT diagnostics**

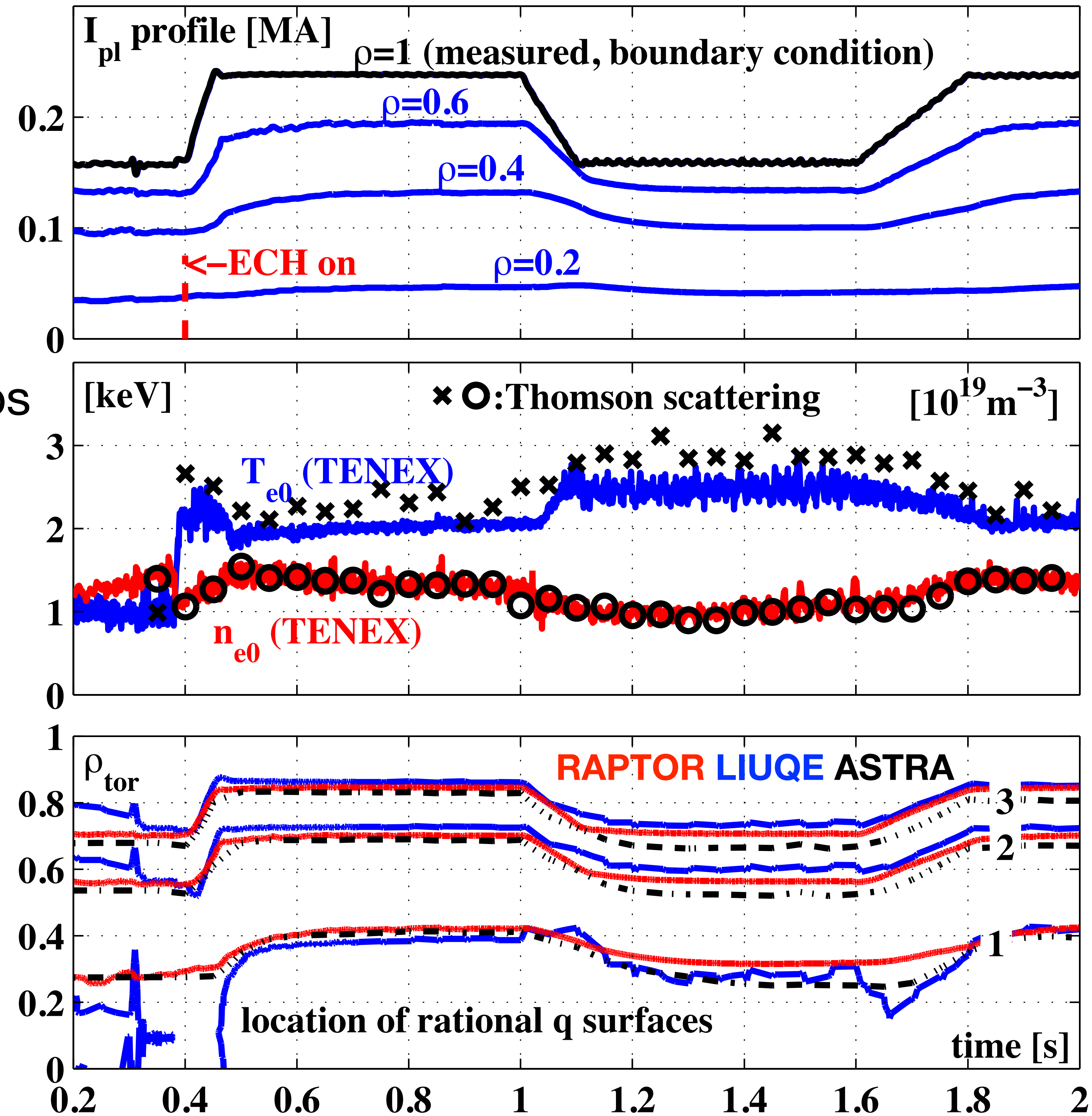
Pilot implementation on TCV demonstrates that real-time simulation is certainly feasible on larger tokamaks

- Current density profile: hard to measure, physics well understood
  - Solve flux diffusion equation with kinetic profiles from real-time diagnostics
  - Flux profile simulated on TCV every 1ms (<150ms current redistribution time)
  - Results comparable to off-line interpretative modeling (ASTRA)



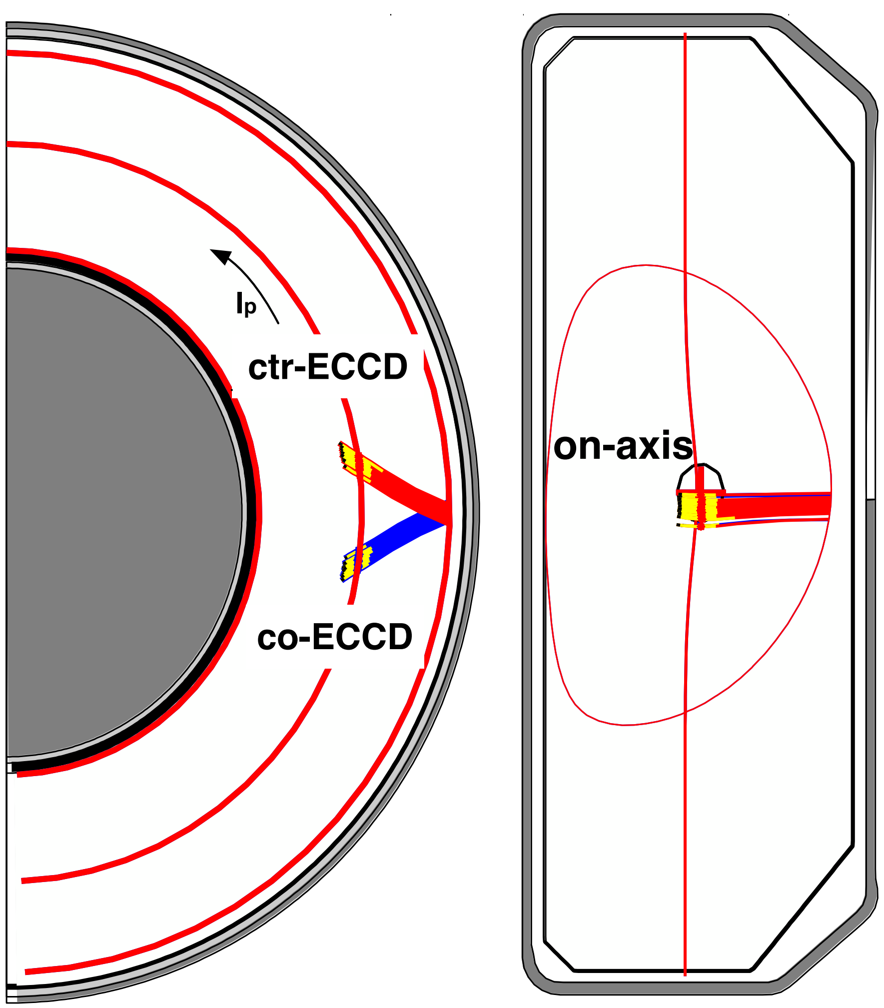
Experiments confirm that RT-RAPTOR gives results similar to off-line estimates

- Experiment involving I_p ramps
  - Compare RAPTOR with
  - LIUQE (offline GS solver)
  - ASTRA (offline interpretative transport modeling)



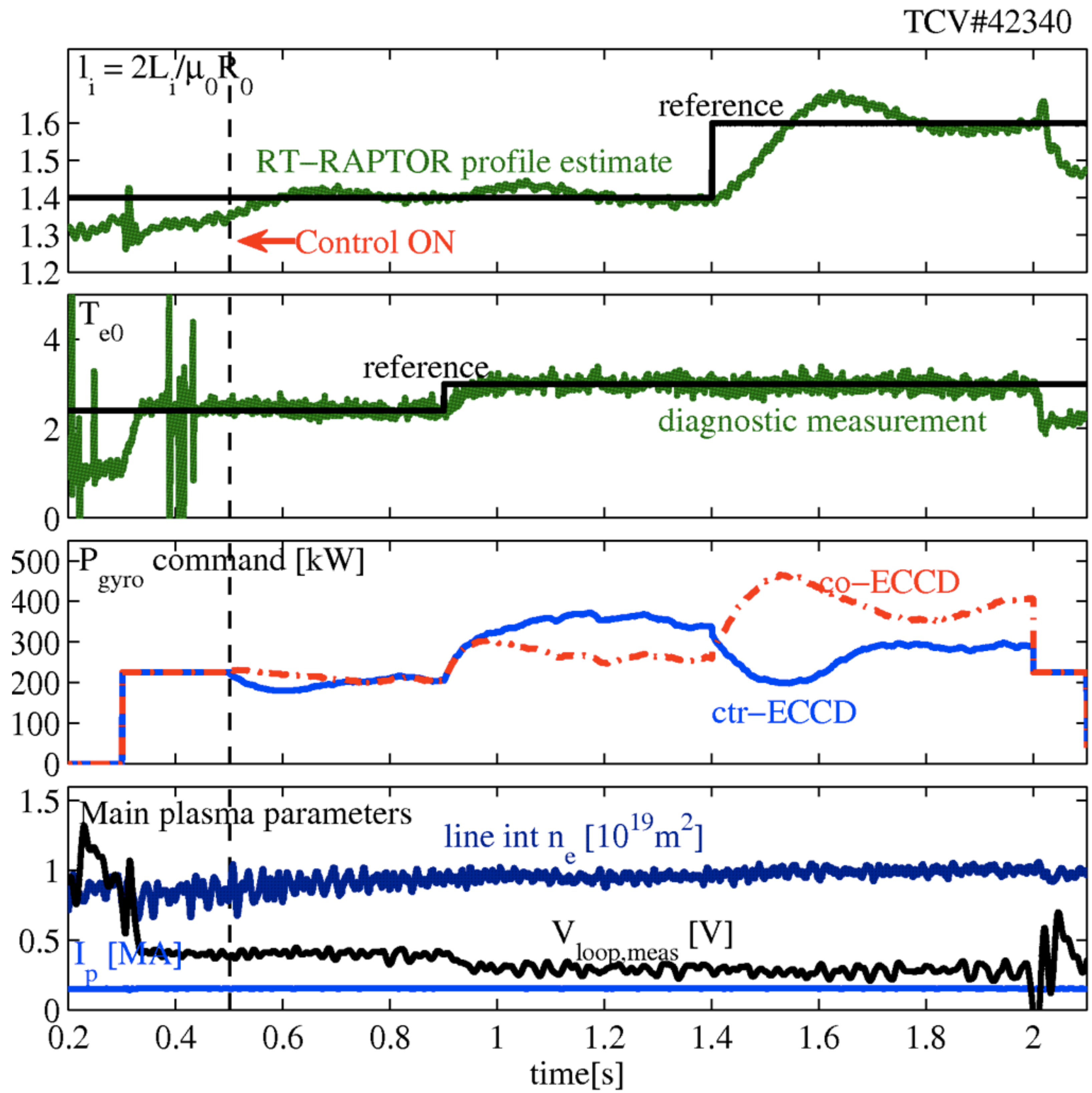
Closing the loop:
Simultaneous feedback control of l_i and T_{e0}

- 1 co-ECCD gyrotron
- 1 counter ECCD gyotron
  - Sum of powers -> T_e
  - Difference of powers -> L_i



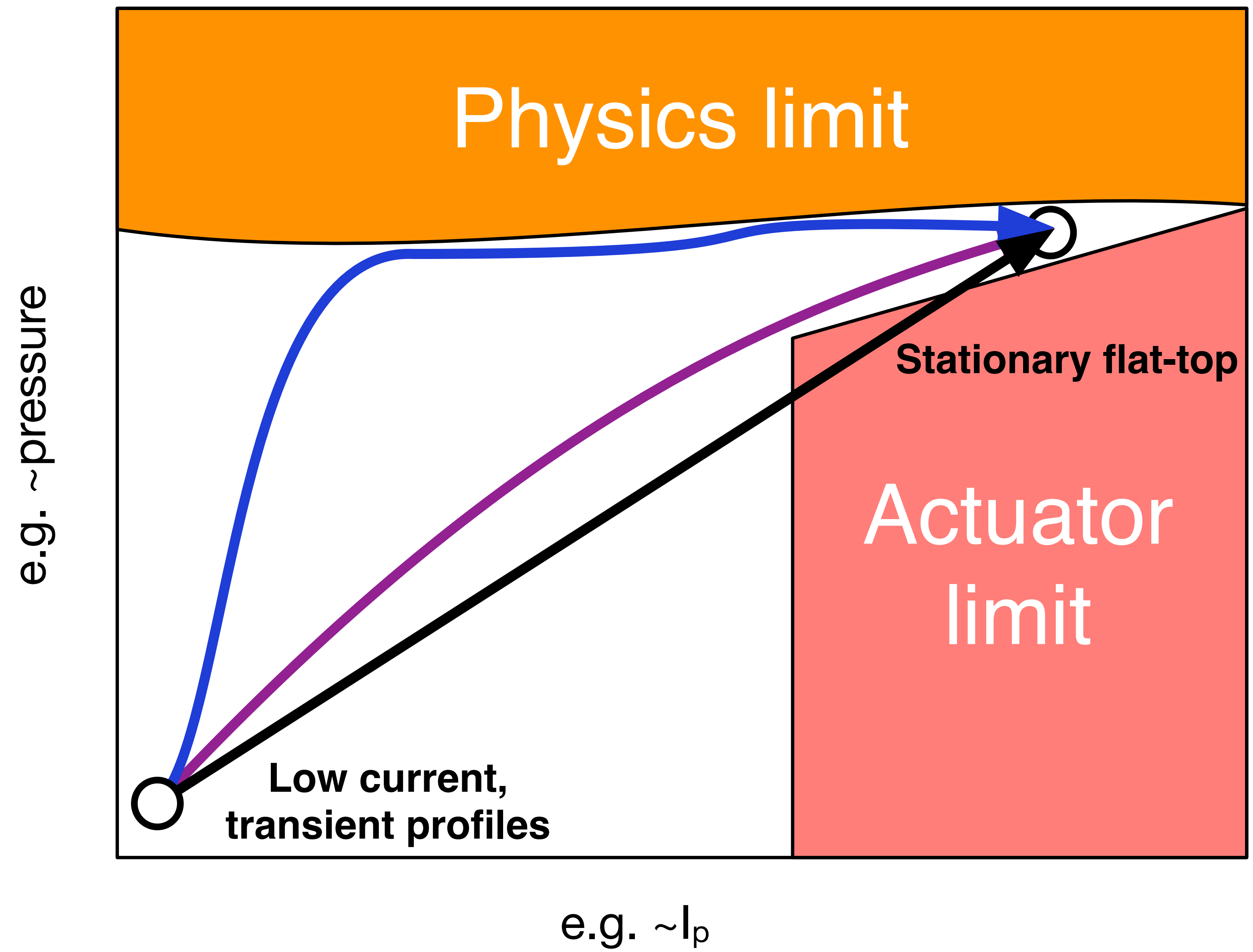
Top view

Side view



Optimization of open-loop trajectories

Tokamak operational space
Which route to take?



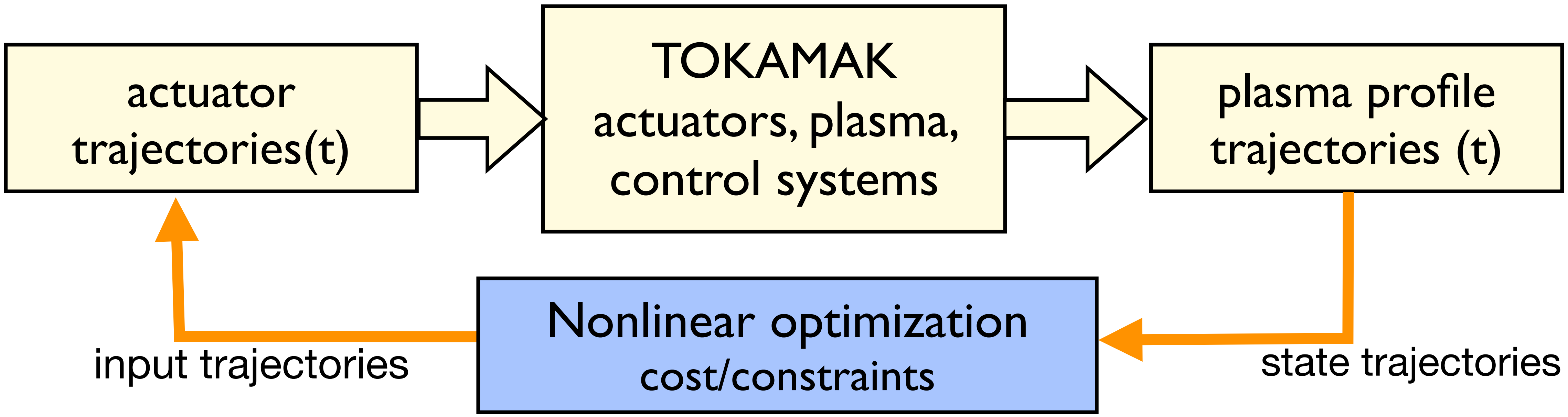
Determine open-loop actuator trajectories: a nonlinear optimization problem

Determine open-loop actuator trajectories: a nonlinear optimization problem

- Given the initial plasma profiles, what input trajectories should I use to:
  - Minimize a **cost function** depending on the *final* plasma state
  - While satisfying **constraints on the state**
  - And satisfying **constraints on the actuators**

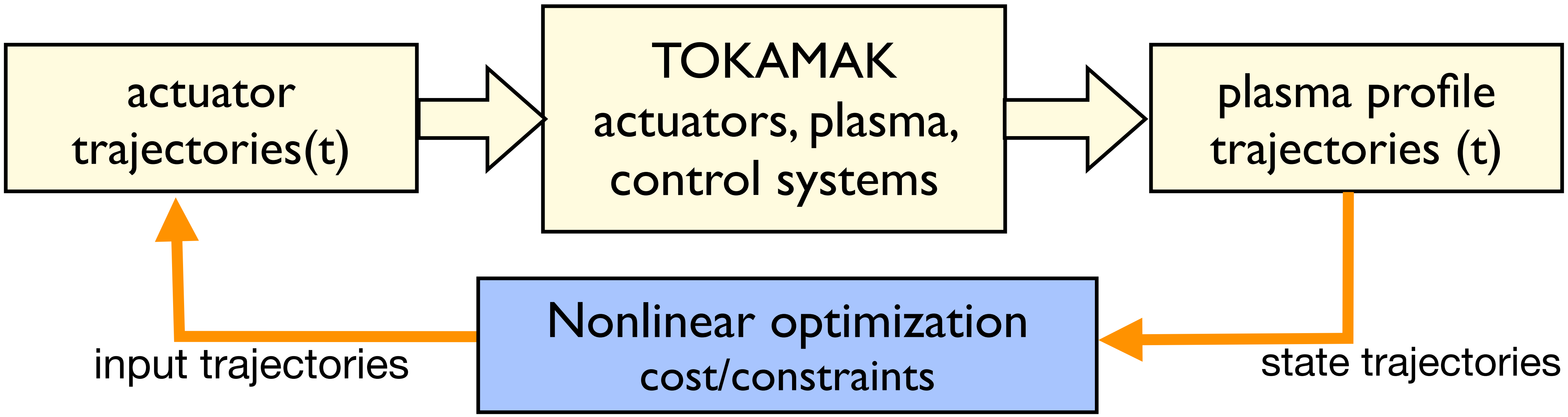
Determine open-loop actuator trajectories: a nonlinear optimization problem

- Given the initial plasma profiles, what input trajectories should I use to:
  - Minimize a **cost function** depending on the *final* plasma state
  - While satisfying **constraints on the state**
  - And satisfying **constraints on the actuators**



Determine open-loop actuator trajectories: a nonlinear optimization problem

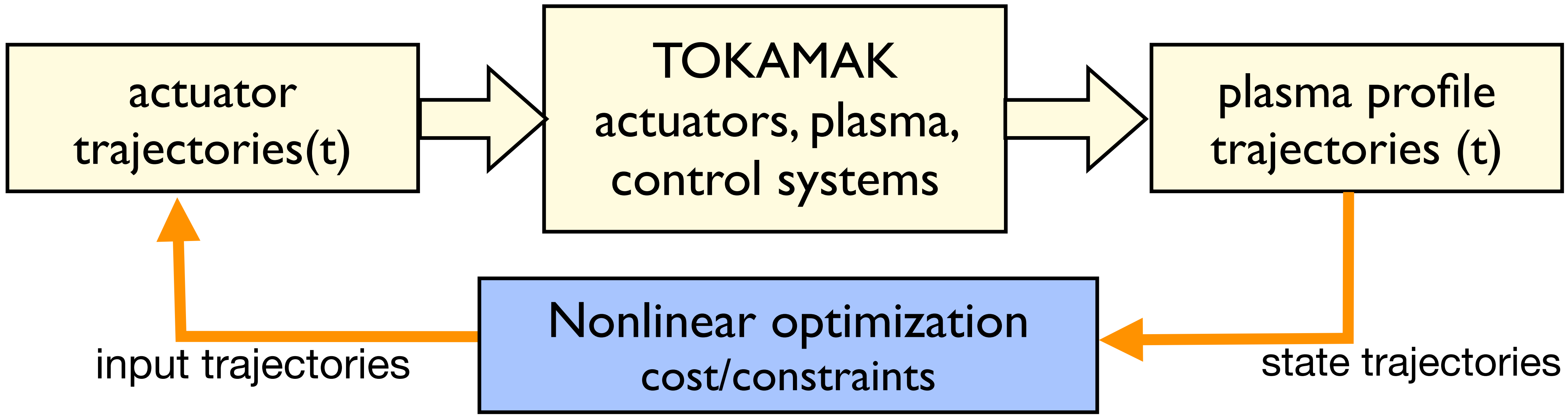
- Given the initial plasma profiles, what input trajectories should I use to:
  - Minimize a **cost function** depending on the *final* plasma state
  - While satisfying **constraints on the state**
  - And satisfying **constraints on the actuators**



- Need to do many simulations

Determine open-loop actuator trajectories: a nonlinear optimization problem

- Given the initial plasma profiles, what input trajectories should I use to:
  - Minimize a **cost function** depending on the *final* plasma state
  - While satisfying **constraints on the state**
  - And satisfying **constraints on the actuators**



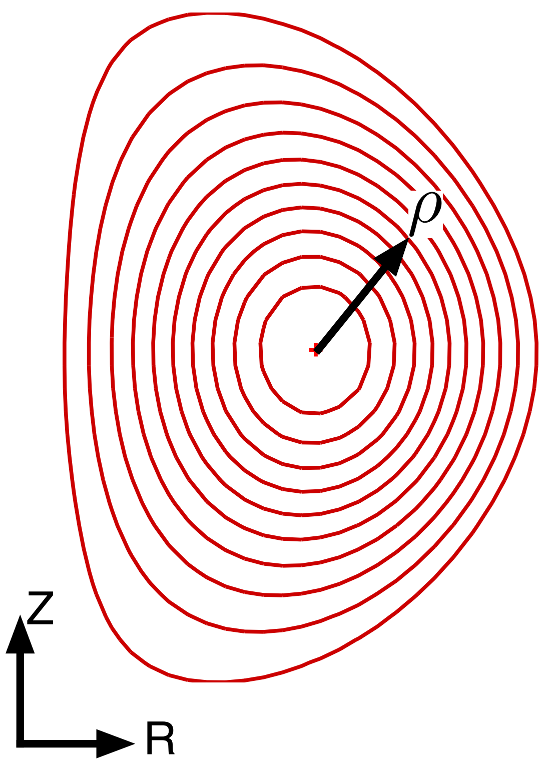
- Need to do many simulations
- Predictive-RAPTOR transport code
  - Returns all gradients of state trajectories w.r.t input trajectory parameters
  - Very fast (one time step: ~10ms, one simulation: ~1 second, full optimization ~1 minute)

Predictive-RAPTOR equations

- Noncircular, axisymmetric, fixed toroidal flux surface shape
- 1D, (flux surface averaged) diffusion of poloidal flux

$$\sigma_{||}\frac{\partial\psi}{\partial t} = \frac{R_0 J^2}{\mu_0\rho}\frac{\partial}{\partial\rho}\left(\frac{G_2}{J}\frac{\partial\psi}{\partial\rho}\right) - \frac{V'}{2\pi\rho}(j_{BS}+j_{ext})$$

  - Neoclassical conductivity ~$T_e^{3/2}$
  - Bootstrap current ~ ∇T_e
  - Current drive sources as sums of gaussians
  - Boundary condition through total I_p



- Flux surface averaged electron temperature diffusion

$$V'\frac{\partial}{\partial t}[n_e T_e] = \frac{\partial}{\partial\rho} n_e\chi_e\frac{\partial T_e}{\partial\rho} + V' P_e$$

  - Fixed ion and density profile
  - Heat sources as sums of gaussians
  - Ad-hoc model for thermal diffusivity

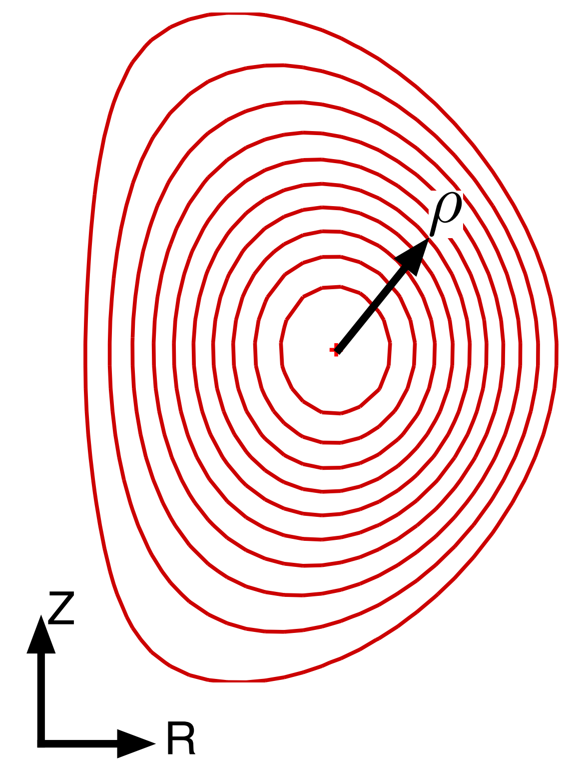
$$\chi_e = \chi_{neo} + c_{ano}\rho q F(s) + \chi_{central} e^{-\rho^2/0.1^2}$$

Predictive-RAPTOR equations

- Noncircular, axisymmetric, fixed toroidal flux surface shape
- 1D, (flux surface averaged) diffusion of poloidal flux

$$\sigma_{||}\frac{\partial\psi}{\partial t} = \frac{R_0 J^2}{\mu_0\rho}\frac{\partial}{\partial\rho}\left(\frac{G_2}{J}\frac{\partial\psi}{\partial\rho}\right) - \frac{V'}{2\pi\rho}(j_{BS}+j_{ext})$$

  - Neoclassical conductivity ~$T_e^{3/2}$
  - Bootstrap current ~ ∇T_e
  - Current drive sources as sums of gaussians
  - Boundary condition through total I_p



- Flux surface averaged electron temperature diffusion

$$V'\frac{\partial}{\partial t}[n_e T_e] = \frac{\partial}{\partial\rho} n_e\chi_e\frac{\partial T_e}{\partial\rho} + V'P_e$$

  - Fixed ion and density profile
  - Heat sources as sums of gaussians
  - Ad-hoc model for thermal diffusivity

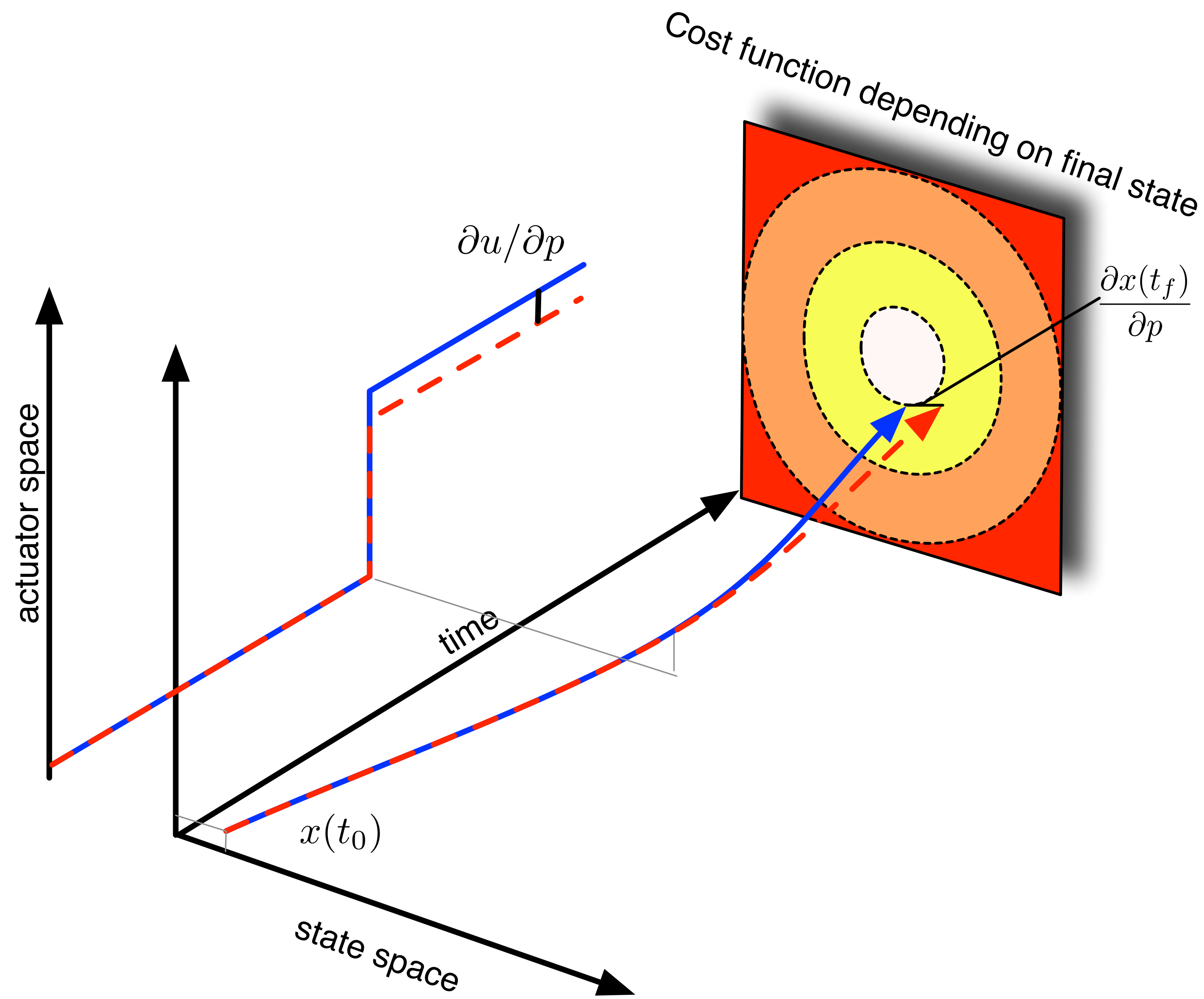
$$\chi_e = \chi_{neo} + c_{ano}\rho q F(s) + \chi_{central} e^{-\rho^2/0.1^2}$$

Compared to other transport models (e.g. CRONOS/ASTRA):

- No self-consistent equilibrium, fixed bnd
- No consistent ray tracing/NBI modules
- No ion or density simulation
- No complex transport models (eg GLF23)

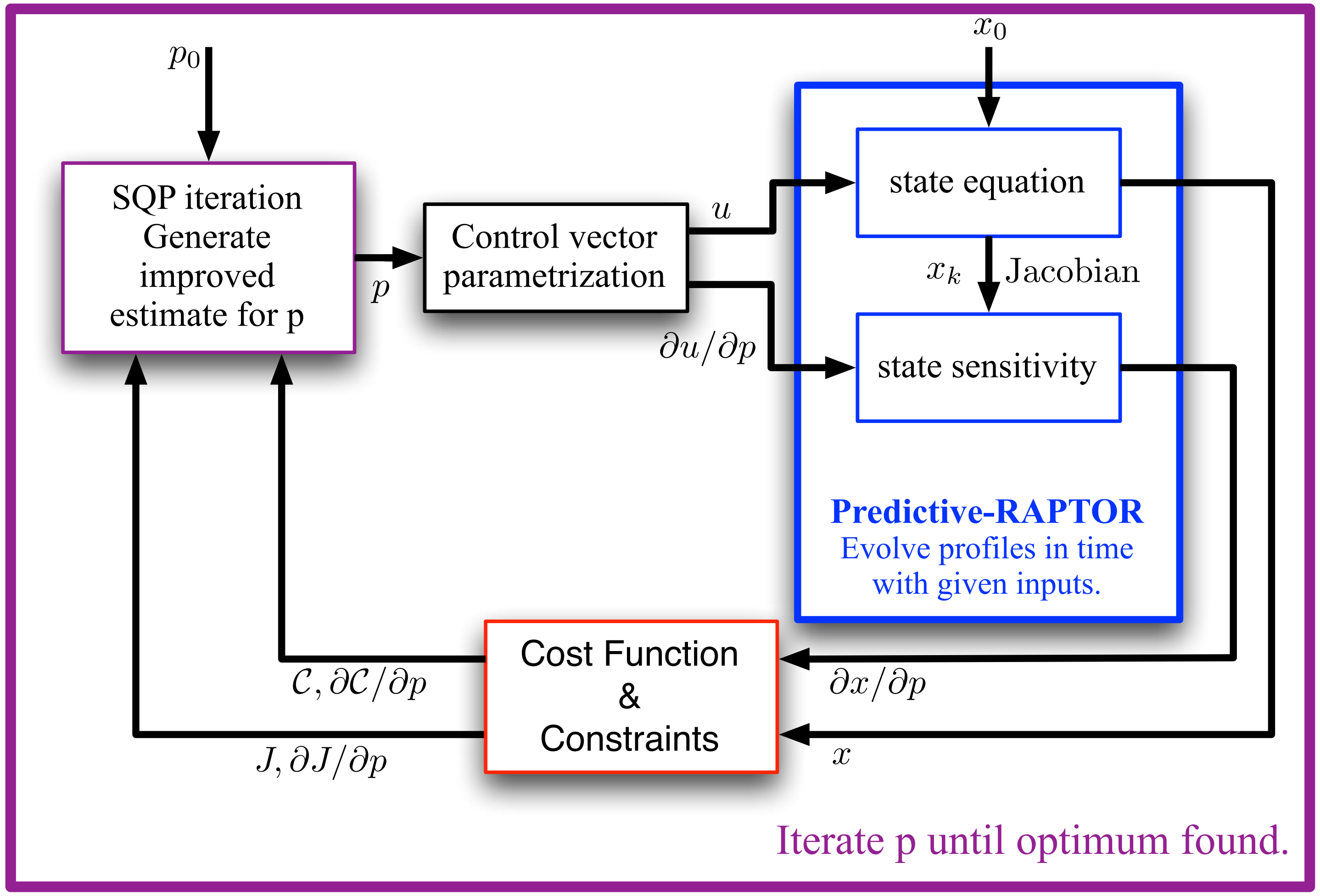
RAPTOR returns gradients of profile evolution w.r.t. input trajectory parameters

- Gradients computed using forwards sensitivity method
  - State sensitivities: dx/dp at all times.
  - Used to quickly evaluate cost function gradient $dJ/dp = dJ/dx_f \, dx_f/dp$
  - Linearization of the profile dynamics around the profile trajectory
    - Useful for control



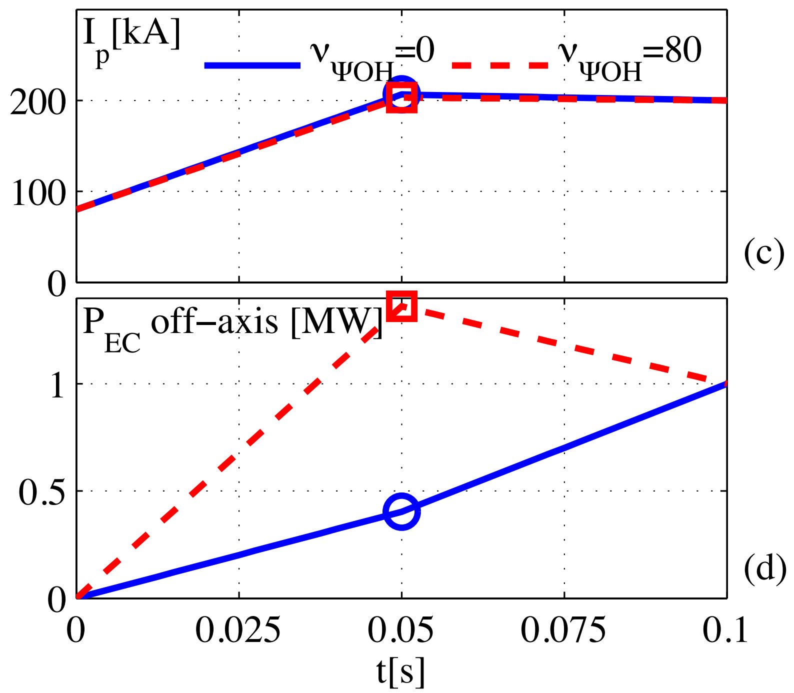
Solution of optimal control problem by SQP nonlinear programming algorithm

- Solution: **Sequential Quadratic Programming (SQP)**
  - Iteratively solve local approximation to nonlinear optimization problem:
    - Quadratic cost function + linear constraints
    - Gradients *dJ/dp* and *dC/dp*, are computed from **state sensitivities**
    - Quasi-newton method for Hessian
    - **Avoid finite-difference evaluation of gradients (expensive!)**
    - Use version implemented in Matlab, called via fmincon



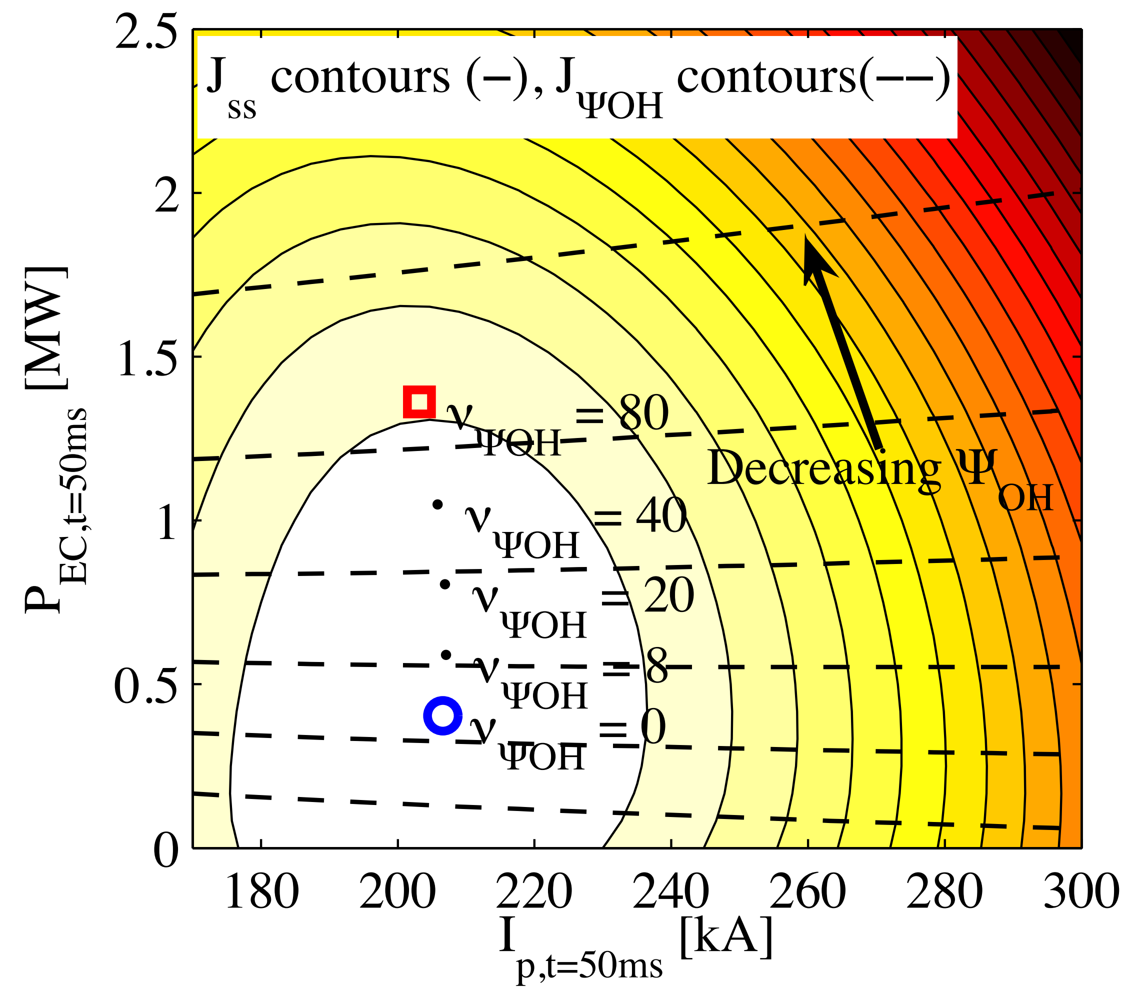
A first example: 2 parameters

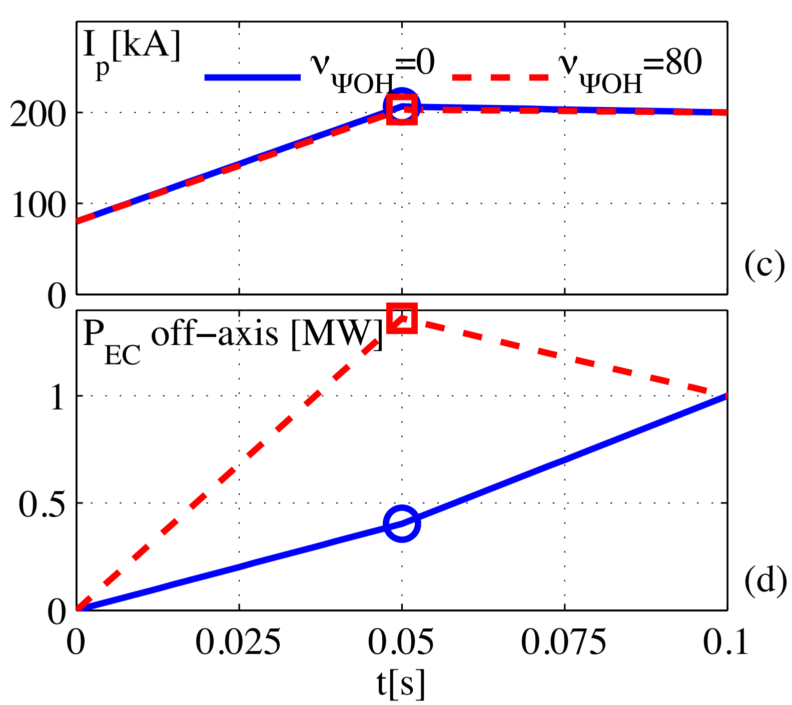
- Ramp up from 80kA to 200kA in 100ms
  - Off axis ECCD at ρ=0.3
- Cost function terms:
  - J_{ss} : Stationary profiles at final time (flat V_{loop})
  - $J_{\Psi OH}$: Flux consumption
  - Total $J = J_{ss} + \nu_{\Psi OH} J_{\Psi OH}$
- 2 parameters: I_p and P_{EC} at t=50ms



A first example: 2 parameters

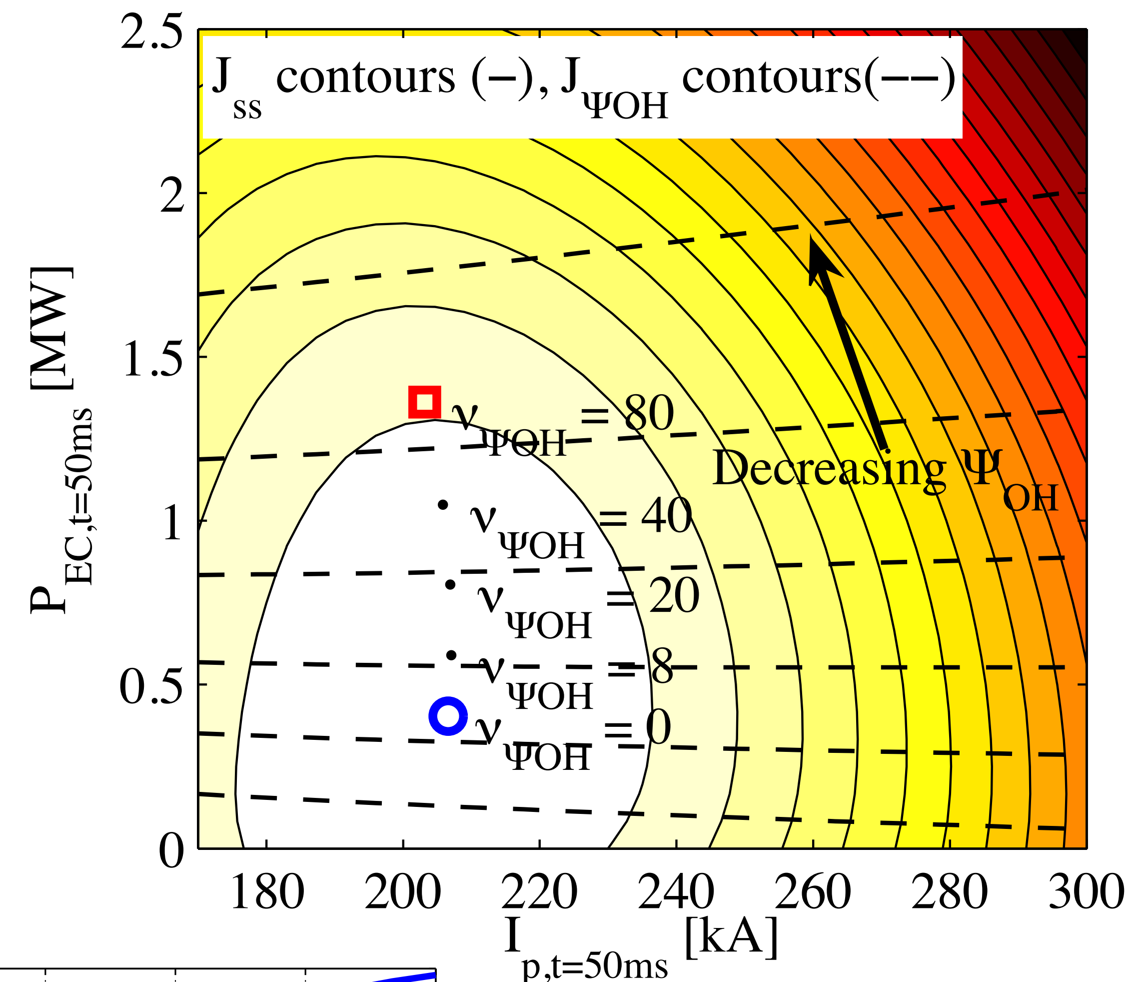
- Ramp up from 80kA to 200kA in 100ms
  - Off axis ECCD at ρ=0.3
- Cost function terms:
  - J_{ss} : Stationary profiles at final time (flat V_{loop})
  - $J_{\Psi OH}$: Flux consumption
  - Total $J = J_{ss} + \nu_{\Psi OH} J_{\Psi OH}$
- 2 parameters: I_p and P_{EC} at t=50ms

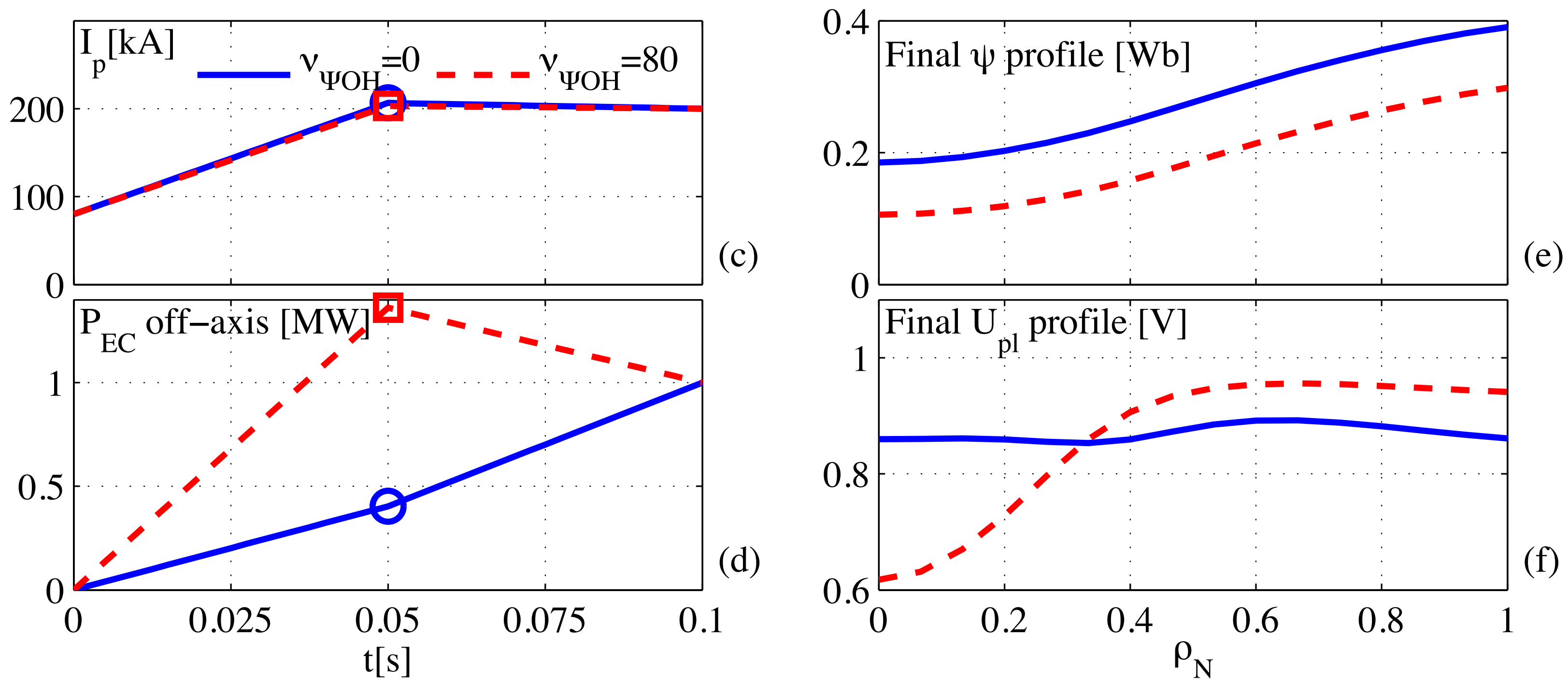




A first example: 2 parameters

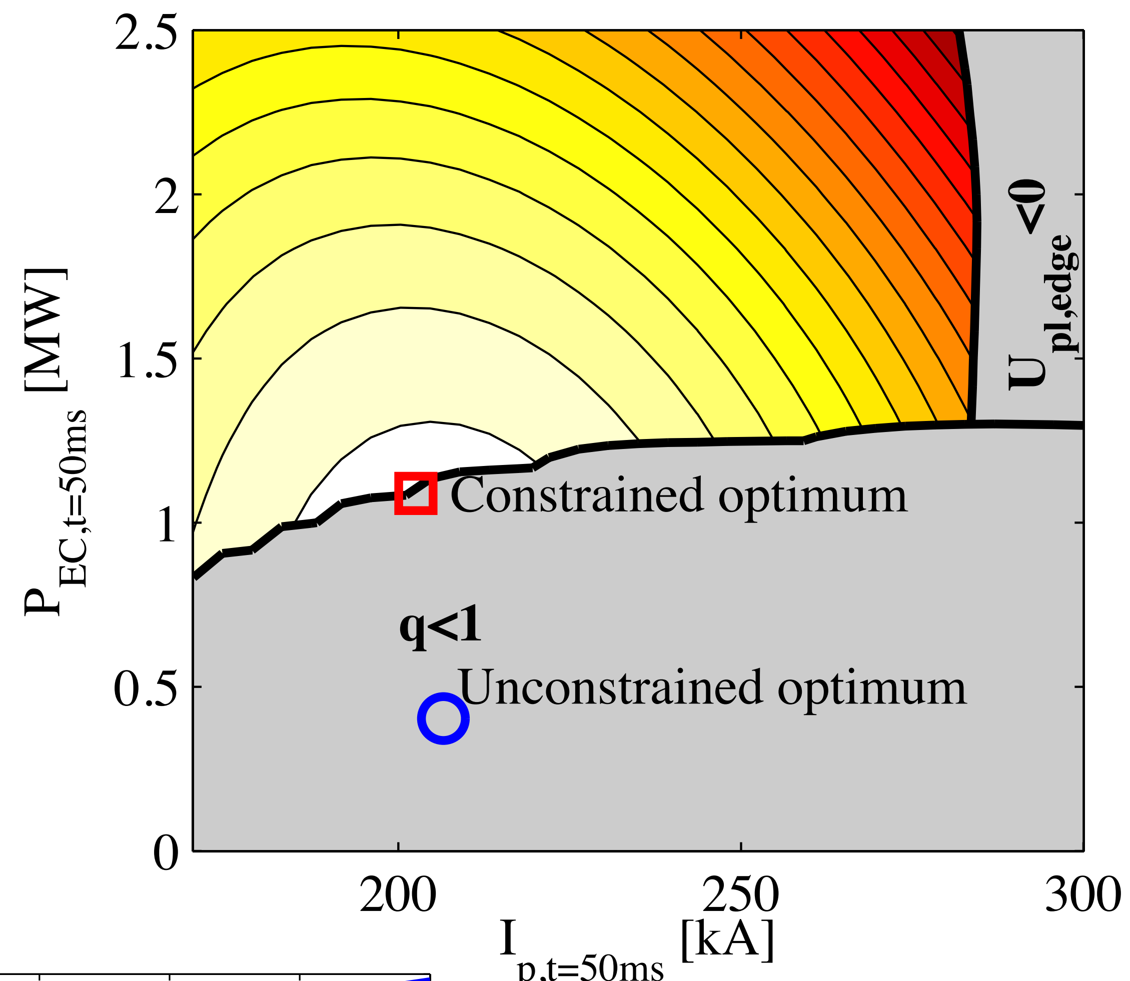
- Ramp up from 80kA to 200kA in 100ms
  - Off axis ECCD at ρ=0.3
- Cost function terms:
  - J_{ss} : Stationary profiles at final time (flat V_{loop})
  - $J_{\Psi OH}$: Flux consumption
  - Total $J = J_{ss} + \nu_{\Psi OH} J_{\Psi OH}$
- 2 parameters: I_p and P_{EC} at t=50ms

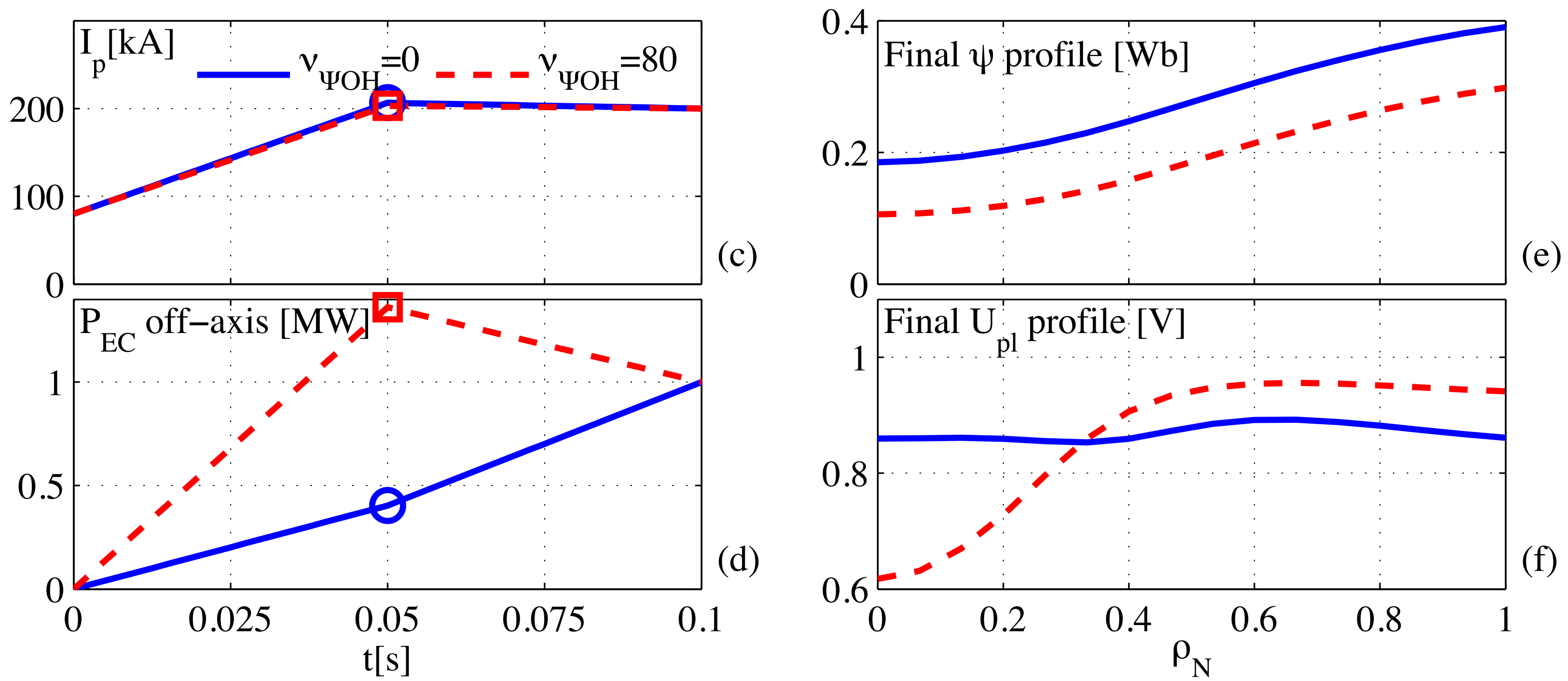




A first example: 2 parameters

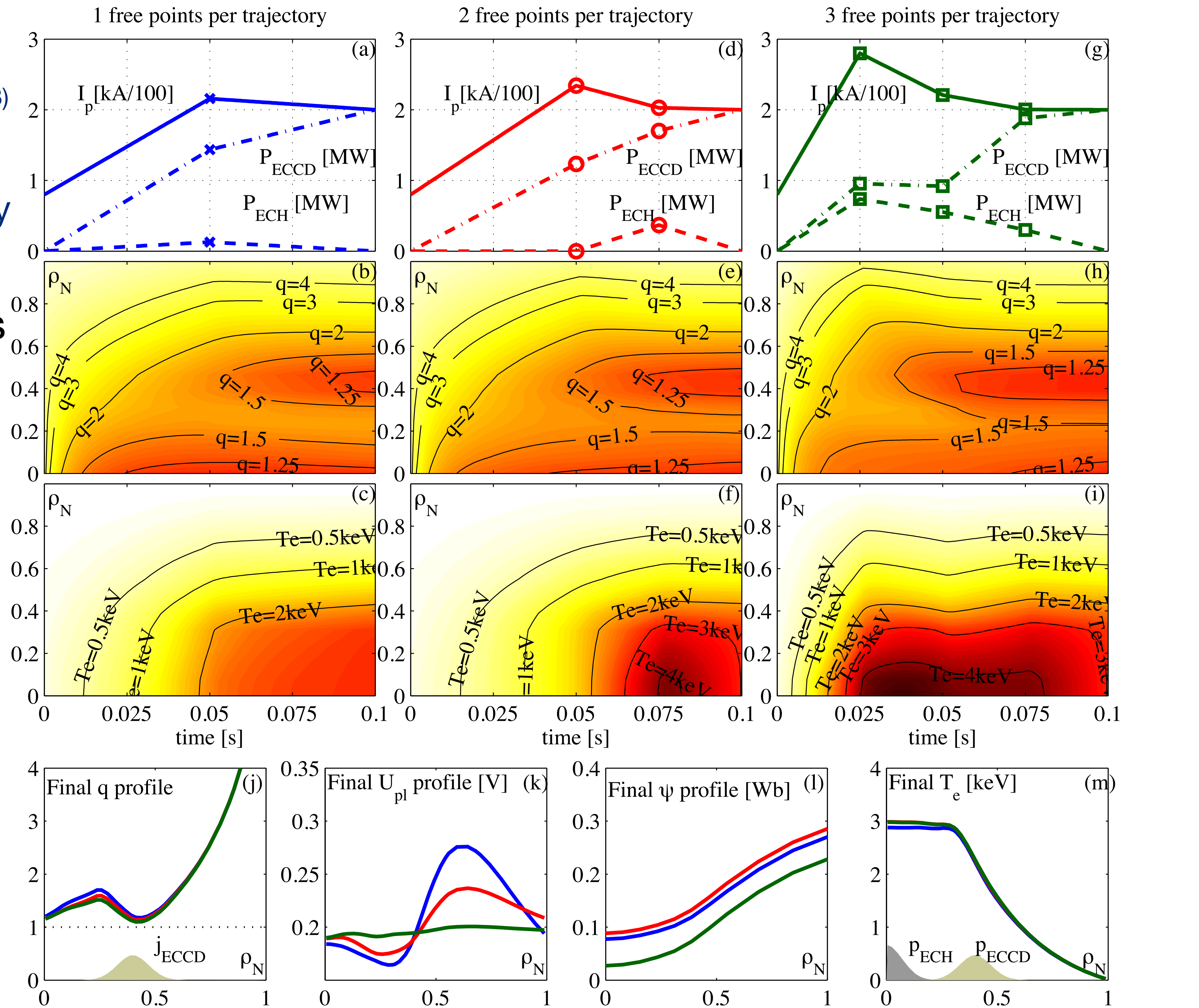
- Ramp up from 80kA to 200kA in 100ms
  - Off axis ECCD at ρ=0.3
- Cost function terms:
  - J_{ss} : Stationary profiles at final time (flat V_{loop})
  - $J_{\Psi OH}$: Flux consumption
  - Total $J = J_{ss} + \nu_{\Psi OH} J_{\Psi OH}$
- 2 parameters: I_p and P_{EC} at t=50ms



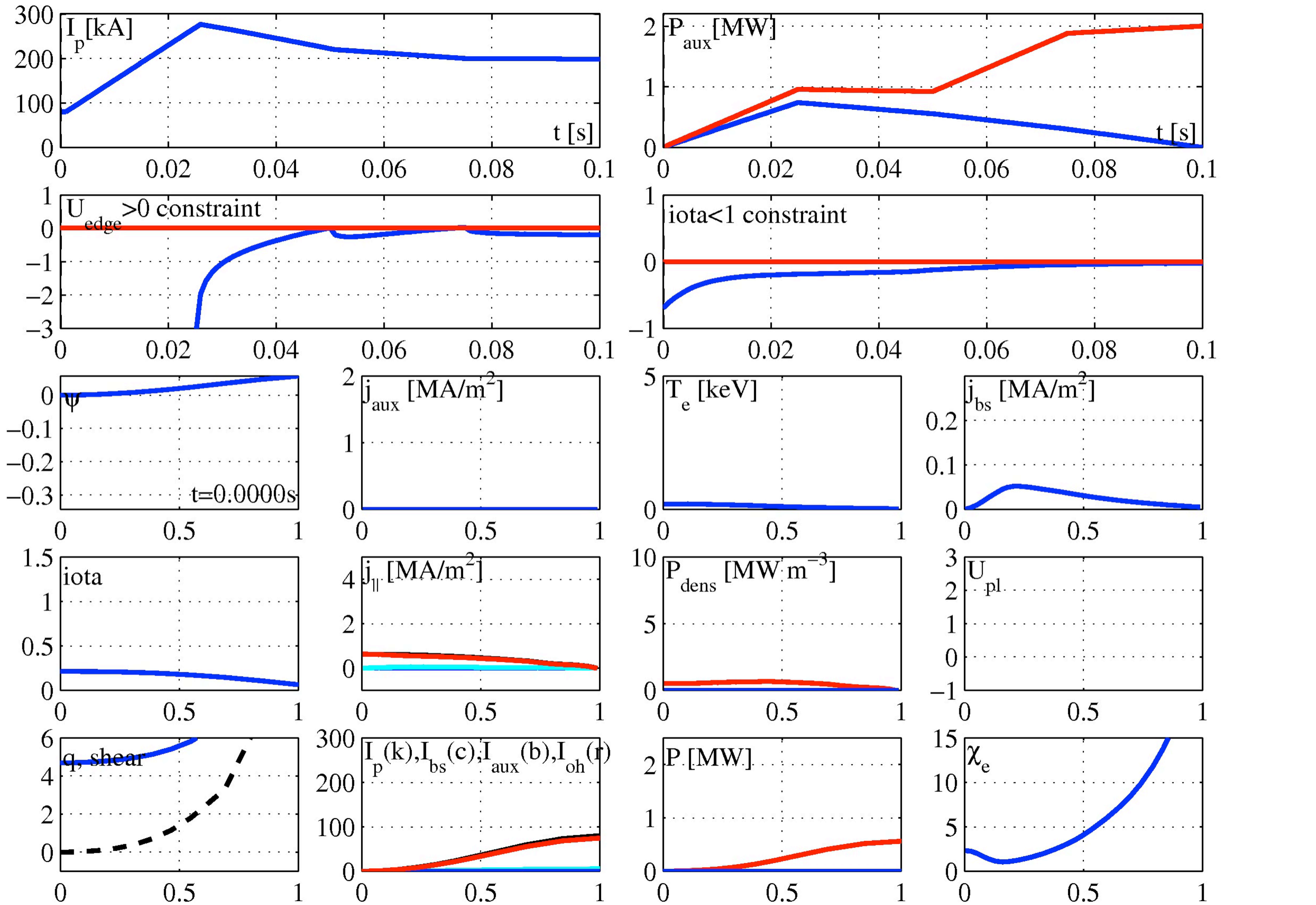


Results for ramp up to 'hybrid' q profile

- Three actuators
  - I_p, $P_{EC(\rho=0)}$, $P_{ECCD(\rho=0.3)}$
- Two cost terms
  - Maximize stationarity
  - Minimize flux cons.
- Multiple constraints
  - $q>1.1$, $V_{loop}>0$
  - dI_p/dt = 8MA/s
  - P_{max} = 2MW, P_{min} =0
- Optimal strategy
  - I_p overshoot
  - Early broad heating
  - Not too much early CD

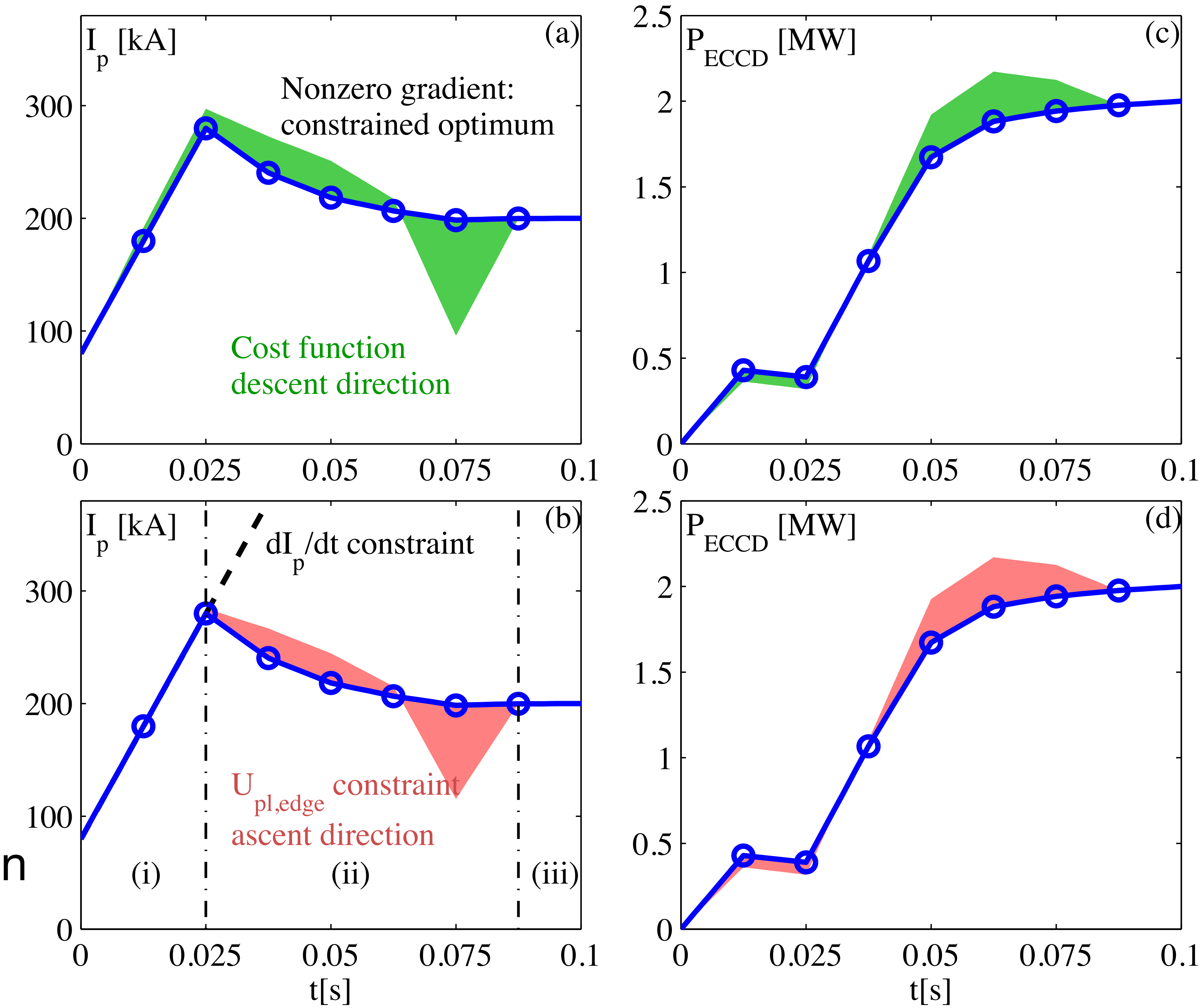


Time evolution with optimal trajectories shows effect of actuator time evolution in flattening final v_{loop} profile



What we can learn from cost & constraint gradients

- Similar scenario, only $U_{pl,edge}>0$ constraint
- Cost function gradient
  - Move in this direction to decrease cost
- Constraint gradient
  - Move in this direction to violate constraint



- Input arc classification
  - (i) Input constrained
  - (ii) State constrained
  - (iii)Unconstrained
- Consequences for feedback control design

Conclusions & Outlook

- New lightweight transport code RAPTOR for physics-based profile control

- Real-time simulation of q profiles demonstrated on TCV
  - q profiles every 1ms, without internal diagnostics!
  - Used for feedback control of T_e and L_i
  - Outlook
    - Closed loop control of q profile in advanced scenarios
    - Integrate with RT current density diagnostics
    - Couple to RT-equilibrium solver

- Optimization of actuator trajectories
  - Actuator trajectories tailored to get stationary profiles at start of flat-top.
  - Outlook
    - Further scenario optimization studies (advanced scenarios, ramp-down)
    - Add more physics (T_i, density, alpha profiles)
    - Real-time prediction

- Application to other tokamaks envisaged - collaborations are welcome

Thank you

Backup slides

Advanced scenario experiments are known to benefit from early I_p overshoot

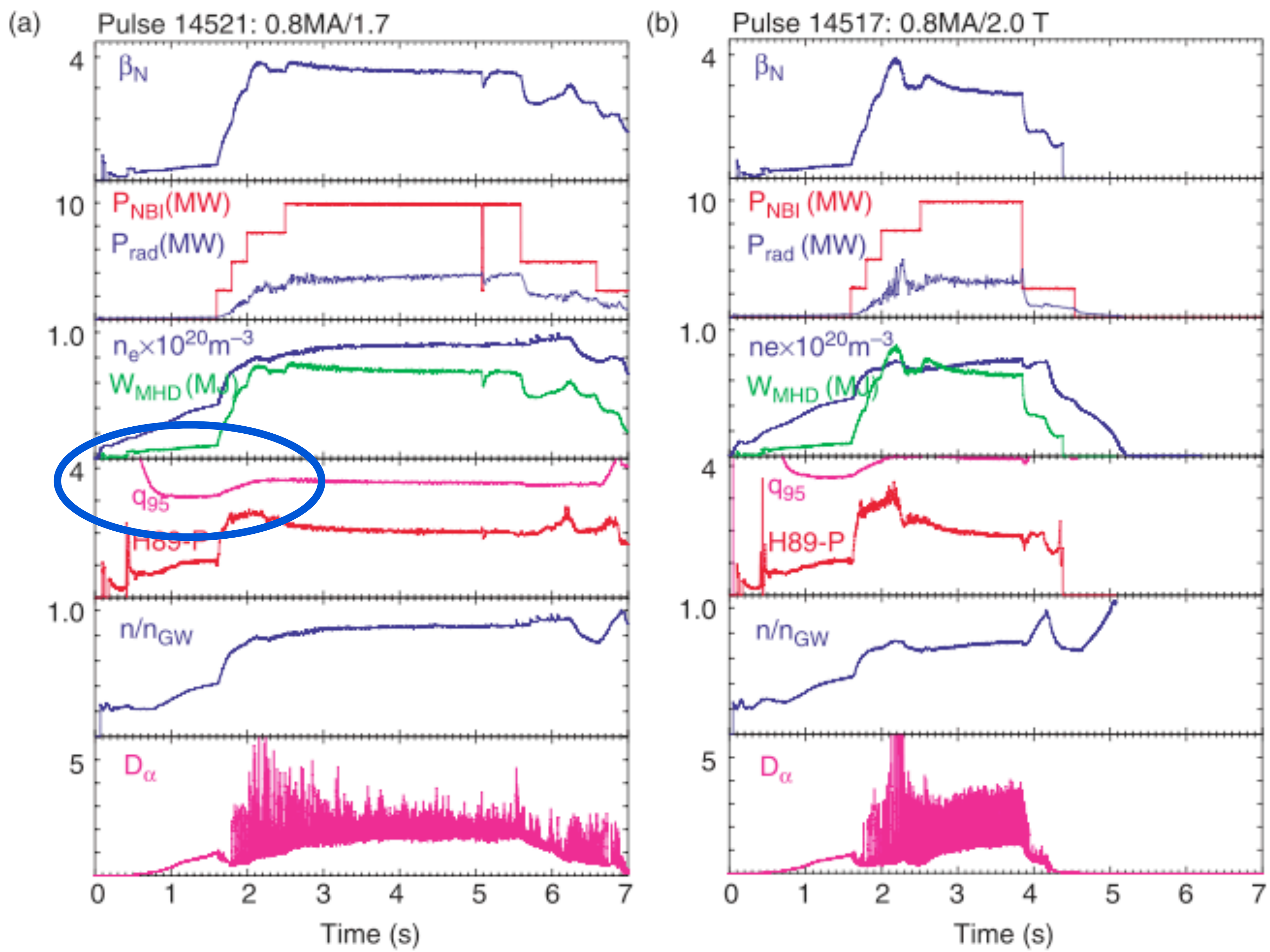
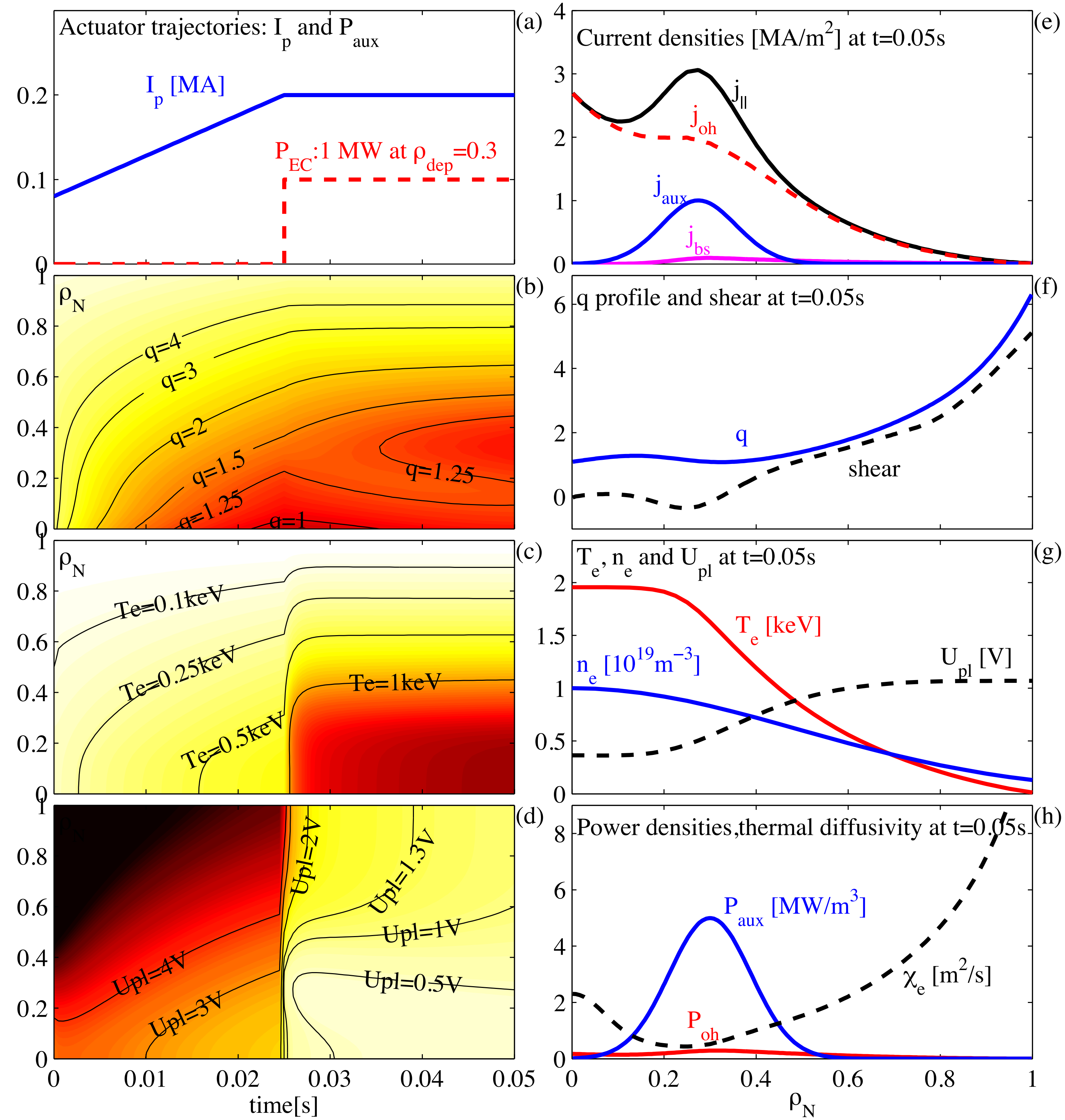


Figure 2. Two discharges that reach high β in ASDEX. (*a*) Pulse 14521 and (*b*) pulse 14517.

[Sips et al, *Progress towards steady-state advanced scenarios in ASDEX Upgrade,* PPCF 2002]

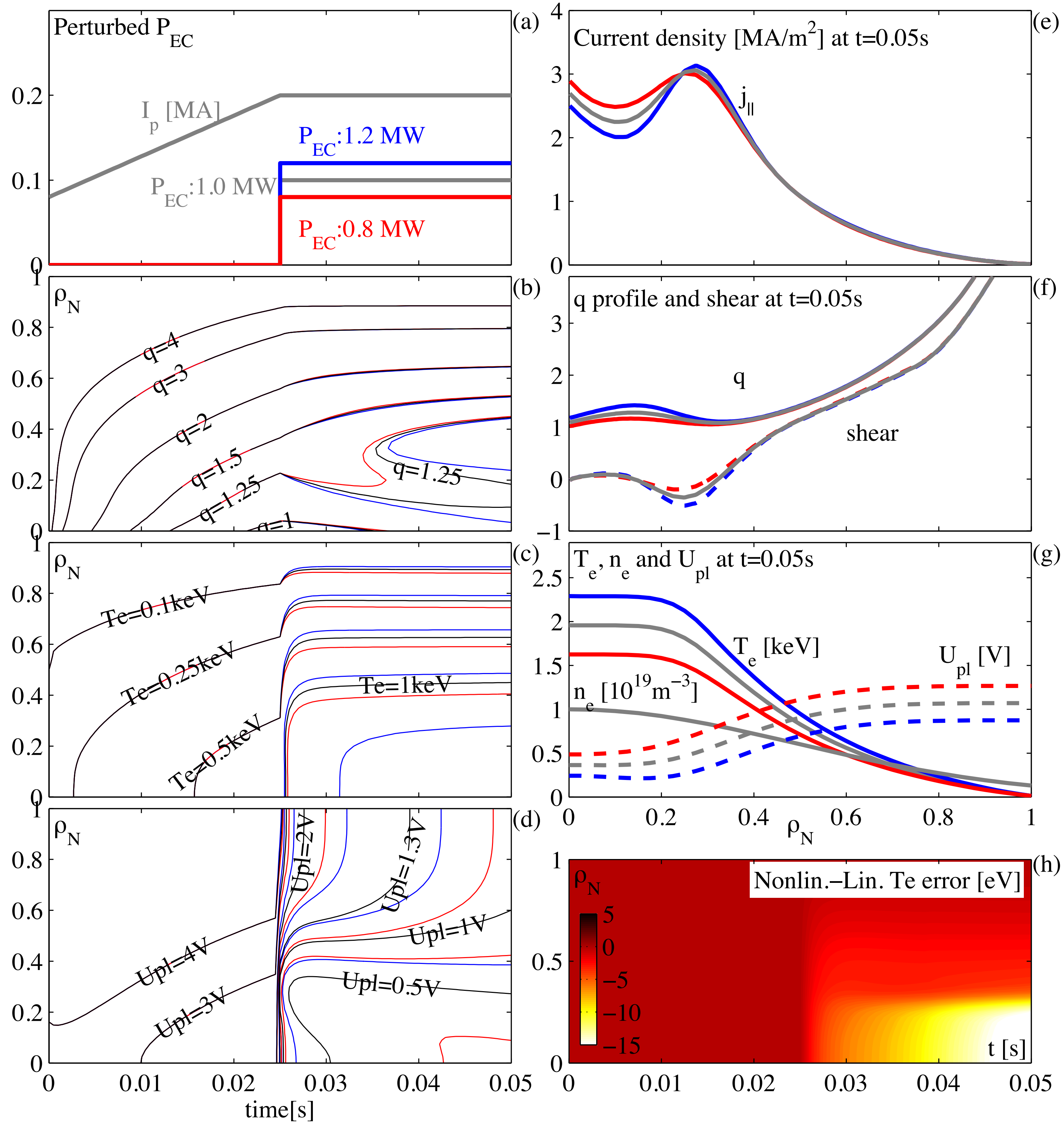
Example of TCV plasma simulation

- Simulation parameters
  - Equilibrium: existing shot
  - Transport model parameters: hand-picked to get reasonable profiles
- Ramp-up scenario
  - 80 to 200kA in 25ms
  - Sudden P_{EC} switch-on
- Some features of the profile evolution:
  - ~zero central shear profile
  - Transient, non-flat U_{pl} profile
  - Improved confinement at low magnetic shear
  - Low j_{BS} contribution
  - Back EMF at j_{CD} location



Evolution of perturbed trajectories

- Parameter: P_{EC} after 0.25
  - 1.2MW
  - 1MW (nominal)
  - 0.8MW
- Perturbed trajectories computed *without running new simulation*
  - Small error w.r.t. nonlinear case



Optimization problem: ingredients

- Cost function: reflects desired properties of **final profiles**
  - Weighted sum of several profile terms
    - 1/safety factor $||1/q(t_f)-1/q_{,ref}||^2$ (e.g. for ITBs)
    - Loop voltage (e.g. for non-inductive scenarios) $||U_{pl}(t_f)-U_{pl,ref}||^2$
    - Loop voltage derivative (for steady-state) $||dU_{pl}/d\rho||^2$
    - Flux consumption (for longer pulse) $||\Delta\Psi_{OH}||^2$
    - Temperature (e.g. for high beta) $||T_e(t_f)-T_{e,ref}||^2$

$$J = \nu_\iota J_\iota + \nu_{U_{pl}} J_{U_{pl}} + \nu_{ss} J_{ss} + \nu_{OH} J_{OH} + \nu_{T_e} J_{T_e}$$

- Constraints: impose limitations on actuator and plasma evolution
  - Constrain current ramp rate, maximum/minimum auxiliary power...
  - Constrain minimum q: $q>q_{min}$ to avoid (e.g.) sawteeth
  - Constrain edge loop voltage: $V_{loop}>0$ to avoid negative edge currents
  - Other constraints possible: shear, j_0, ...

- Parametrize u(t) with a finite number of parameters *p* using basis functions *P(t)*

$$u_i(t) = \sum_j^{n_i} P_{ij}(t) p_{i,j}$$

Implicit numerical scheme for time-evolution

- Given state x_k, inputs u_k at time step k,
  - PDE(ρ,t) -> discretize -> Nonlinear ODE at each time step:

$$\tilde{f}(x_{k+1}, x_k, u_k) = \tilde{f}_k = 0 \,\forall\, k$$

  - Take steps in Newton descent direction *d* $\mathcal{J}_{k+1}^{k} d = \tilde{f}_k,$
    - Need Jacobian $$\mathcal{J}_{k+1}^{k} = \frac{\partial \tilde{f}_k}{\partial x_{k+1}}$$
    - Obtained from analytical expression for all the derivatives (copious application of chain rule)
  - Iterate until residual f_k < tolerance
  - Go to next time step
- Store Jacobians at each time step

Parameter sensitivity of profile evolution

- Time evolution depends on mode parameters
  - One example: a transport model parameter
  - Another example: a parameter defining the input trajectory

$$\tilde{f}(x_{k+1}, x_k, u_k) = \tilde{f}_k = 0 \;\forall\; k$$

- Differentiating with respect to parameter *p*, we get the *sensitivity equation*

$$0 = \frac{\mathrm{d}\tilde{f}_k}{\mathrm{d}p} = \frac{\partial \tilde{f}_k}{\partial x_{k+1}} \frac{\partial x_{k+1}}{\partial p} + \frac{\partial \tilde{f}_k}{\partial x_k} \frac{\partial x_k}{\partial p} + \frac{\partial \tilde{f}_k}{\partial u_k} \frac{\partial u_k}{\partial p} + \frac{\partial \tilde{f}_k}{\partial p}$$

  - Linear ODE for dx_k/dp, solve while evolving nonlinear PDE: ***Forward sensitivity analysis***
  - Jacobians df_k/dx_k, df_k/dx_{k+1} are known from Newton iterations
  - Computational cost proportional to *p*

- dx_k/dp gives the **linearization** of the state trajectories in the parameter space

$$T_e(\rho, t)|_{p=p_0+\delta p} \approx T_e(\rho, t)_{p_0} + \frac{\partial T_e}{\partial x} \frac{\partial x}{\partial p} \delta p$$

End